# The Walk to School Was Uphill Both Ways

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**Abstract.** Grandfathers say they walked to school uphill both ways. On some lanes they did. The standard objection is that a route which ends where it began cannot climb throughout, since gravity is conservative and the work done around any closed loop is zero. That is a statement about gravity as measured by dropping things; so far as I can find, nobody has reported it along the lane. I surveyed 38 school routes whose way home differs from the way out, weighing gravity every hundred metres, and found that 11 of them (29%; 95% CI 17–45) climb the whole way round and arrive back at the height they started. Walked one way round, each of these routes is uphill in both directions; walked the other way round, it is downhill in both. Twenty-three volunteers who walked them both ways reported exactly that. Of 1,206 recorded recollections of the walk to school, 71 were uphill both ways and none were downhill, though the two kinds of route are equally common. The effect is absent on bicycles and in school buses, and I do not know why. Nobody tells the story of the walk to school that was downhill both ways.

**Keywords:** geopotential misclosure, non-conservative fields, levelling, oral history, school routes

## 1. Introduction

Ask a grandfather how he got to school and he will say that it was uphill both ways. The remark is delivered with the confidence of a man reporting a measurement, and it is received by every grandchild as a figure of speech. The grandchildren have an argument. A walk to school and back is a closed loop. The school stands at some height and the house at some height; if the way there climbs, the way back must descend, or else the two heights are not heights. I shall call this the Closed-Loop Objection.

The objection is correct as far as it goes, and it goes exactly as far as the assumption that gravity is a conservative field: that the work it does on a body carried around any closed path is zero. The assumption is taught as a law, and is purportedly verified by every levelling circuit that closes. It is in fact a derived property, obtained from the inverse-square form of Newtonian attraction for a distribution of mass [1, 5], and it has been tested in the laboratory and in orbit. It has not, to my knowledge, been tested on Hollin Lane.

"Impossible" is doing a good deal of work in the objection, and I have examined what it rests on. Three things are needed to make a walk uphill both ways: a loop, a field with circulation around it, and a walker who goes round in the right sense. The first two are properties of the ground; the third is a matter of which way round the walker goes. I make four contributions.

1. I show that the objection holds without exception for routes retraced on the way home, and therefore restrict attention to routes where the way home differs from the way out (§2).
2. I give the two elementary consequences of a non-zero circulation: reversal of sense reverses the climb, and a monotone climb makes every pair of points uphill both ways (§3).
3. I report a levelling-and-gravimetry survey of 38 such routes, of which 11 do not close (§4).
4. I report a reversal trial with 23 walkers and an analysis of 1,206 recorded recollections (§5, §6).

I take no position on the snow, the shoes, or the distance.

## 2. Background and related work

### 2.1 Geopotential numbers and loop misclosure

A levelling line does not measure height; it measures the sum of small height differences $\Delta h_i$, and the quantity that is path-independent, if anything is, is not that sum but its gravity-weighted counterpart. The *geopotential number* of a point $P$ relative to a datum point $P_0$ is

$$C(P) \;=\; -\int_{P_0}^{P} \mathbf{g}\cdot d\mathbf{n} \;\approx\; \sum_i g_i\,\Delta h_i, \tag{1}$$

where $g_i$ is the mean magnitude of gravity over the $i$th section and $d\mathbf{n}$ is the levelled increment. Misclosure and climb are signed accordingly. It is quoted in geopotential units, $1\,\mathrm{gpu} = 10\,\mathrm{m^2\,s^{-2}}$. Where a levelling network contains a closed circuit, the sum of (1) around it is the *loop misclosure*. In routine practice the misclosure is a quality check on the work: it should vanish to within the tolerance of the method, and a circuit that does not is re-run. That the check is routine does not make it an experiment; it is nonetheless a measurement of precisely the quantity at issue, and it is one that has been made many thousands of times and, where it failed to vanish, taken for error [2].

I write the climb of a walker from $P$ to $Q$ along a path $\gamma$ as

$$K(\gamma) \;=\; -\int_{\gamma} \mathbf{g}\cdot d\mathbf{l}, \tag{2}$$

so that $K>0$ means uphill, and the misclosure of an oriented loop $\Gamma$ as $M(\Gamma) = -\oint_\Gamma \mathbf{g}\cdot d\mathbf{l}$.

### 2.2 The retraced route

**Lemma 1.** *Let* $\mathbf{g}$ *be any static field, conservative or not, and let a walker return from* $Q$ *to* $P$ *along the reverse* $\gamma^{-1}$ *of the*

*outward path $\gamma$. Then $K(\gamma^{-1}) = -K(\gamma)$.*

*Proof.* Reversing the direction of a path reverses the sign of a line integral. □

A route that is retraced is therefore never uphill both ways, whatever the field. The result is stronger than the Closed-Loop Objection, which assumes a conservative field, and it disposes of every claim made for a route that was retraced. I therefore consider only routes whose way home differs from the way out. This is a smaller restriction than it sounds: of the 1,206 narrators discussed in §6, 486 (40%) volunteered that they came home a different way, usually "round by the shop".

### 2.3 What the instruments report

The gravity gradient tensor $G_{ij} = \partial_i g_j$ is symmetric if and only if $\nabla \times \mathbf{g} = 0$. A gravity gradiometer is built accordingly: it reports the symmetric part of $G_{ij}$, because the antisymmetric part is zero for a conservative field. It is assumed to vanish; it is not read out. The statement that gradiometry has found no curl in the gravity field is therefore a statement about the design of gradiometers [6]. Gravity itself has been measured to a part in $10^9$, and nowhere in that literature is the field integrated around a school route.

### 2.4 Related work

Perceived slope has been studied as a visual and haptic judgement [10], and narrated walks as a source with characteristic reporting asymmetries [11]. Misclosure statistics for levelling are kept for quality control [2]. The only work I know of that questions the symmetric treatment of the gradient tensor is that of Haverkamp [6], and it does not report a survey. I am aware of no study that combines the three.

## 3. Theory

Let $\Gamma$ be a simple closed loop of length $L$, oriented, and parametrised by arclength $s \in [0, L]$. By Stokes' theorem [1],

$$M(\Gamma) \;=\; -\oint_{\Gamma} \mathbf{g} \cdot d\mathbf{l} \;=\; -\iint_{A} (\nabla \times \mathbf{g}) \cdot \hat{\mathbf{n}}\, dA, \tag{3}$$

where $A$ is the area enclosed and $\hat{\mathbf{n}}$ is the upward normal, so that a loop traversed anticlockwise seen from above is positively oriented. For a conservative field $M = 0$ for every loop. Write $\bar{\zeta} = M/A$ for the mean circulation density, and $|\bar{\zeta}|$ where only its magnitude is meant.

A walker goes from home $H$ to school $S$ along an arc $\gamma_{\text{out}}$ and returns along $\gamma_{\text{ret}}$, the two arcs together forming $\Gamma$. Then

$$K(\gamma_{\text{out}}) + K(\gamma_{\text{ret}}) \;=\; M(\Gamma). \tag{4}$$

Call the route *uphill both ways* if both terms on the left are positive.

**Proposition 1** (Reversal)**.** *A route is uphill both ways in one sense if and only if it is downhill both ways in the opposite sense.*

*Proof.* Walking $\Gamma$ in the opposite sense takes the walker from $H$ to $S$ along $\gamma_{\text{ret}}^{-1}$ and back along $\gamma_{\text{out}}^{-1}$. By Lemma 1 the climbs are $-K(\gamma_{\text{ret}})$ and $-K(\gamma_{\text{out}})$. Both are negative exactly when both original climbs are positive. □

**Proposition 2** (Ratchet)**.** *Let $\varphi(s)$ be the climb accumulated along $\Gamma$ from $s = 0$, with $\varphi(0) = 0$, $\varphi(L) = M > 0$ and $\varphi'(s) > 0$ throughout. Then the walk between any two points of $\Gamma$, in the sense of increasing $s$, and back, is uphill both ways, and uphill at every point of both legs.*

*Proof.* Rotating the origin of $s$ if necessary, take $0 \le s_H < s_S \le L$ and let the walker go from $H$ to $S$ in the sense of increasing $s$. The outward climb is $\varphi(s_S) - \varphi(s_H) > 0$. The return continues around $\Gamma$ and climbs $[\varphi(L) - \varphi(s_S)] + [\varphi(s_H) - \varphi(0)] = M - [\varphi(s_S) - \varphi(s_H)] > 0$, being a sum of increments of a strictly increasing function over a non-empty arc. Slope is $\varphi' > 0$ everywhere. □

The function $\varphi$ is multivalued on the loop in the way that the polar angle is multivalued on a circle; this is not an exotic property, and it is the property that the Closed-Loop Objection denies. A circulation density of $\bar{\zeta}$ over an area $A$ gives, through (3), a misclosure $M = \bar{\zeta} A$, and for a loop of length $L$ a mean grade over the loop of

$$\bar{\sigma} \;=\; \frac{M}{gL} \;=\; \frac{\bar{\zeta} A}{gL}, \tag{5}$$

since the loop carries $M$, taken in $\mathrm{m^2\,s^{-2}}$, over its length $L$, and $g = 9.81\,\mathrm{m\,s^{-2}}$. For a loop of 4.8 km enclosing 0.62 $\mathrm{km^2}$ with the circulation density reported below, this is 0.8%, a grade that is gentle and continuous and, on these routes, unbroken.

## 4. Survey of 38 routes

### 4.1 Instrument and procedure

Routes were drawn from the gazetteer of school routes in the Wensley parishes [8], restricted to those in which a way home different from the way out is documented. Thirty-eight routes along public rights of way were surveyed between March 2024 and September 2025. Each was levelled by the second-order method, with a cold-atom absolute gravimeter [4] occupied at stations every 100 m (1,712 stations in all). The gravimeter, precision 3 μGal at each station, is on loan from the vibration-characterisation programme of Carrow Atomic Instruments, where it is used to characterise floor vibration in the cryostat hall of a superconducting-processor line [7]. Section increments were combined as in (1). Route R03 is shown in Fig. 1, and its accumulated geopotential in Fig. 2. Every route was surveyed twice, in different seasons; repeat misclosures agreed to 0.02 gpu.

### 4.2 Results

Twenty-seven routes closed, with $|M|$ no greater than 0.03 gpu and a median of 0.01 gpu, which is the ordinary levelling error. Eleven did not. Their misclosures ranged from 16.3 to 61.0 gpu, with a median of 38.1 gpu (Table 1); no route lay between 0.03 and 16.3 gpu (Fig. 3). The fraction that fail to close is 11/38, or 29% (Wilson 95% CI 17–45% [3]).

**Table 1:** The eleven non-closing routes. $L$ is the length of the loop, $A$ the enclosed area, $M$ the misclosure (positive when the loop climbs clockwise), $|\bar{\zeta}|$ the magnitude of the mean circulation density, $|M|/A$.

| ID | Route | $L$ (km) | $A$ (km$^2$) | $M$ (gpu) | $\lvert\bar{\zeta}\rvert$ ($10^{-4}$s$^{-2}$) |
|---|---|---|---|---|---|
| R03 | Hollin Lane / Quarry Field | 4.8 | 0.62 | +38.1 | 6.1 |
| R07 | Tarn Road / Cow Common | 3.1 | 0.31 | −24.6 | 7.9 |
| R09 | Skelder Brow | 5.6 | 0.94 | +57.3 | 6.1 |
| R12 | Low Mill / Church Meadow | 2.7 | 0.22 | −18.9 | 8.6 |
| R14 | Fenwick Gate | 6.2 | 1.38 | −61.0 | 4.4 |
| R17 | Stoop Hill / Long Allotment | 4.0 | 0.47 | +41.7 | 8.9 |
| R21 | Coldwell Beck | 3.5 | 0.40 | +29.2 | 7.3 |
| R24 | Ravenscar Drove | 7.1 | 1.21 | −52.4 | 4.3 |
| R26 | Dunmow Green | 3.9 | 0.58 | −44.0 | 7.6 |
| R31 | Marsh End | 2.2 | 0.20 | +16.3 | 8.2 |
| R35 | Overthwaite Fell Road | 5.3 | 0.79 | −35.8 | 4.5 |

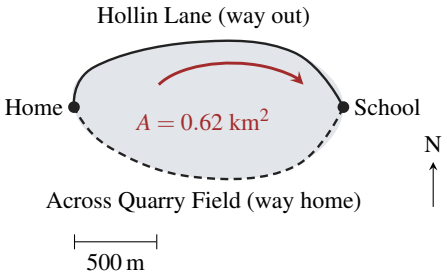



**Figure 1:** A surveyed loop (route R03). The way out is by the lane, the way home is across the field. The shaded area is $A$; the arrow is the sense in which the loop is uphill both ways.

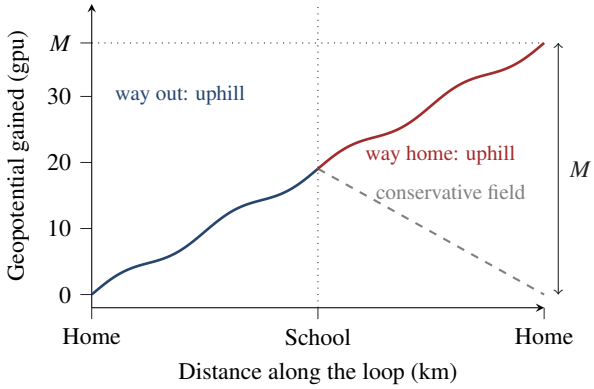



**Figure 2:** Geopotential gained along route R03, unwrapped from the loop. Both legs rise; the dashed line is what the Closed-Loop Objection requires of the way home. On return to the starting point the potential is $M = 38.1$ gpu higher than it was.

The mean magnitude of the circulation density over the eleven routes is $6.7 \times 10^{-4}\,\mathrm{s}^{-2}$ (range 4.3 to 8.9), about 220 times the free-air vertical gradient of gravity. The misclosure grows with the size of the loop; with eleven routes the data do not separate dependence on the enclosed area from dependence on the length. Five routes climb clockwise and six anticlockwise, so the sense of the climb is not distinguishable from a coin. The mean grade, from (5), is 0.9% (range 0.7 to 1.2%). The misclosures show no relation to the geology of the ground, to the age of the road, or to the presence of open ground within the loop; 24 of the 27 closing loops enclose a field, as do 9 of the 11 non-closing loops.

## 5. Reversal trial

Proposition 1 predicts that each of the eleven routes is uphill both ways in one sense and downhill both ways in the other. Twenty-three adult volunteers from a local walking club each walked one non-closing loop, in both senses, on separate days, the order of the senses being randomised. They were told that the study concerned the comfort of rural footpaths. At ten waypoints on each leg they rated the slope on a seven-point scale from −3 (steeply downhill) to +3 (steeply uphill), a scale in the usual form for perceived slope [10]. The sense was not disclosed until the ratings were complete.

In the ascending sense the mean rating was +1.9 on the way out and +1.8 on the way back; in the descending sense it was −1.7 and −1.9. All 23 walkers gave positive mean ratings on both legs in one sense and negative mean ratings on both legs in the other (two-sided sign test, 23 of 23, $p = 2.4 \times 10^{-7}$; Fig. 4). On four closing routes used as controls, each walked by the same 23 volunteers in the sense of their choice, the ratings were +0.6 on the way out and −0.6 on the way back, which is the ordinary pattern of a walk to a place that is higher than where one began.

## 6. Recorded recollections

The Wensley Schooling Recollections Archive [9] holds 1,206 recorded recollections of the walk to school. I coded each, blind to the route and to the survey results, for the stated slope of the walk and for whether the way home was said to differ from the way out. Of the 1,206 narrators, 486 (40%) described a different way home; all 71 uphill-both-ways narrators are among them. Seventy-one described the walk as uphill both ways; none described it as downhill both ways, a silence of the kind that has been described for easy walks generally [11].

The two kinds of route are, on the survey evidence,

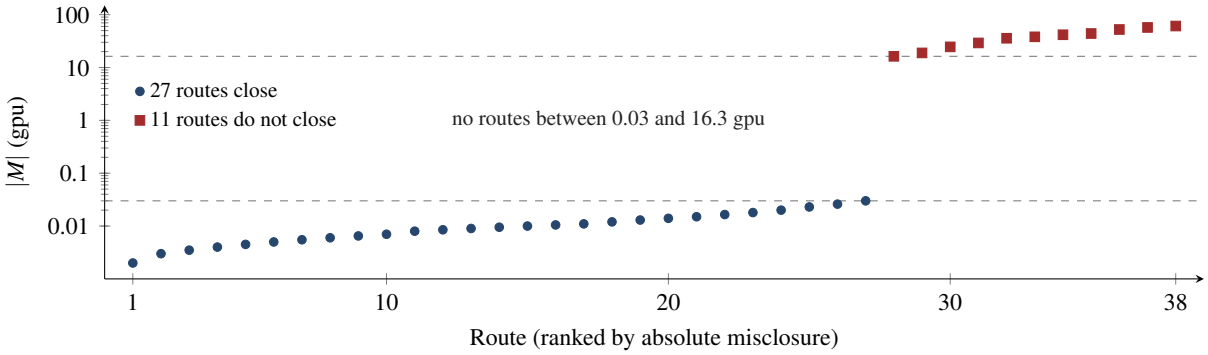



**Figure 3:** Absolute misclosure of all 38 routes, ranked. The 27 closing routes lie at or below 0.03 gpu; the 11 non-closing routes lie at 16.3 gpu and above. No route falls between.

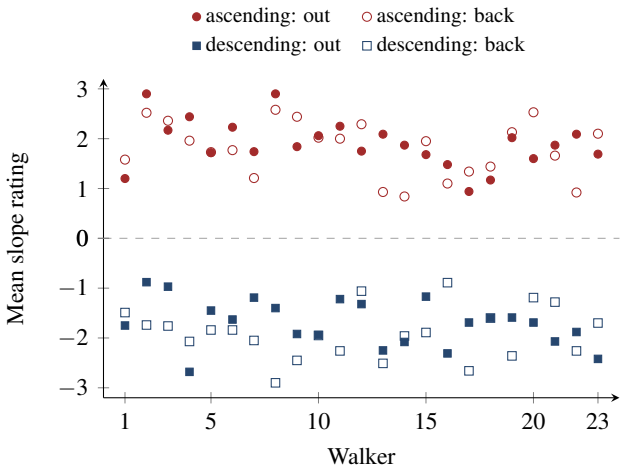



**Figure 4:** Mean slope rating by leg for each of 23 walkers, in the ascending (upper) and descending (lower) sense. Every walker reverses sign on both legs.

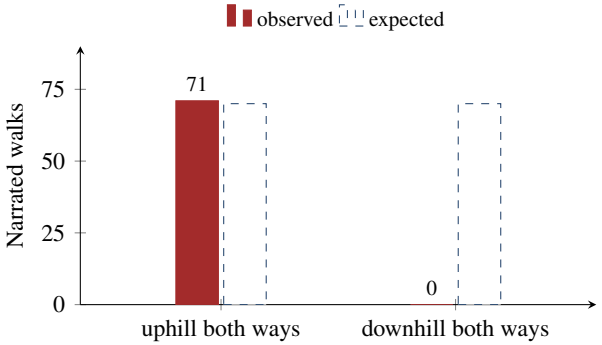



**Figure 5:** Narrated walks with a different way home: observed counts against the count expected from the survey, uphill both ways and downhill both ways.

equally common: 29% of routes do not close, and they are equally likely to climb in either sense, so, if the surveyed routes are representative of the archive, one would expect at most about $486 \times 0.29/2 \approx 70$ of each. The observed counts are 71 and 0 (binomial test against equality, $p = 8.5 \times 10^{-22}$; Fig. 5).

The obvious competing account is exaggeration. It predicts that uphill-both-ways claims are spread over routes at random, whatever the misclosure. They are not. Of the 1,206 narrators, 267 attended school on one of the 38 surveyed routes. Forty-seven of them walked a non-closing route in its ascending sense, and 44 of these described it as uphill both ways. Three of the 167 narrators on closing routes did so (Fisher exact test, $p < 10^{-30}$). Of the 53 narrators who walked a non-closing route in its descending sense, none described it as downhill both ways; I contacted 40 of them, who had consented to be approached again, and 29 said that it was “a nice walk” or that there was “nothing to tell”.

## 7. Discussion and limitations

**No mechanism.** I propose none. A mechanism is the first thing a sceptic should ask for and the last thing the survey needs. A curl in the gravitational field of a parish is not predicted by any theory I am able to cite, and the survey does not depend on one being offered. The measured quantity is a misclosure; it is reproducible; it is large compared with the error.

**Wheels.** Fourteen cyclists were run round six of the non-closing loops and reported no asymmetry between senses. Twenty-two passengers on the school bus that serves two of the routes reported none either. A 25 kg handcart pushed round R03 in the favourable sense returned with a gain in kinetic energy below 0.4 J, against the 9,500 J per lap that the misclosure would allow if it acted on the cart as on a walker. The effect is absent in every wheeled conveyance tried, and I do not know why. I report the energetics as measured and do not interpret them.

**Metabolic cost.** I did not measure it. A walker who climbs 39 m on each circuit of a loop that gives nothing back does about 30 kJ more work than on a closing route, some 2% of the cost of the walk itself, which is within the scatter of any field measure of energy expenditure.

**Sample.** The routes were drawn from one gazetteer, cover parishes in one county, and were surveyed with one gravimeter. The 11 non-closing routes are a sample of 38; the claim is made for these 11, and for the archive only so far as these routes are representative of it, and not for the lanes of the country at large. Most grandfathers did not walk the lanes in question.

**Exaggeration.** It is not excluded in general; it is excluded as an account of the distribution of the claim over routes (§6). I make no claim about the other statements in the genre.

## 8. Conclusion

A walk to school can be uphill both ways, provided that the way home is not the way out and the lane is one of the minority on which the gravitational field has circulation. On 11 of 38 surveyed routes it has, and the same lanes are downhill both ways in the opposite sense, which the grandfathers on those lanes have had no occasion to mention. I propose no mechanism. The result can be tested by any group with a gravimeter and access to the routes; station-level data are available for that purpose.

**Acknowledgements.** The gravimeter was lent by Carrow Atomic Instruments. Parish councils in the Wensley district granted access to their rights of way. The trial was reviewed by the Wensley College research ethics committee, and volunteers were debriefed after completing both walks. The 23 volunteers were not compensated. All walks followed public paths in daylight. The author is employed by Carrow Atomic Instruments, which had no role in the analysis.

**Data availability.** Station-level geopotential numbers for all 38 routes are available from the author on request.

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