The Stairs Have One More Step Than Anyone Has Counted: A Double-Observer Survey of 2,184 Staircases

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Abstract

People descending stairs in the dark sometimes put a foot out for a step that is not there. We report that every staircase has one more step than has ever been counted. Two independent observers, one counting by eye and one by foot, surveyed 2,184 staircases in 311 buildings. The eye count fell one step short of the foot count on 2,141 flights, a mean shortfall of 1.00 steps per staircase (95% CI 0.99–1.01), whether the flight had four treads or thirty-one. A single hidden object would produce a constant shortfall of this kind; a general habit of overlooking steps would produce one that grows with flight length. Illuminance made no difference. The foot registers the extra step within a few descents, and the sensation of a missing step is the moment the two counts are reconciled. We treat each flight as a closed population in which every member but one is seen, and give a correction that building owners can apply by adding one step to the drawn count. The estimator did not converge on spiral staircases, and we did not excavate. The step is real and lies just below the floor.

1 Introduction

Three ordinary observations about stairs concern us:

1. *the lurch*: the foot descends for a tread and meets the floor early;
2. *the count*: a person who has counted a flight aloud on Monday arrives, on Tuesday, a step short of the figure they hold;
3. *the drawing*: the architect's section agrees with the person on Monday and not with the foot on Tuesday.

The literature treats these as three problems in inattention, arithmetic and drafting, respectively [6, 7]. We propose that they are a single problem in *detection*: the eye and the foot are two observers of one population, and they do not return the same census.

We set aside the inherited assumption that the number of steps in a flight is the number of steps a person can see. What remains is a population of treads and two instruments with different detection functions. The question is then empirical: how many treads does each instrument report, and how many are there?

2 Related Work

Three bodies of work are relevant.

1. **Petersen (1896)** marked fish and counted returns, establishing that a population may be estimated by comparing two independent captures [1]. Lincoln (1930) and Chapman (1951) gave the estimator its modern form [2, 3].
2. **Otis et al. (1978)** classified closed populations by the way capture probability varies, including the case of the trap-shy member that no trap takes [4].
3. **Nichols et al. (2000)** brought two observers into the same field and showed that the discrepancy between them is itself a measurement [5].

Whereas those authors counted animals, stair research has counted occasions. Halvorsen and Ruiz [6] recorded 9,400 missteps in public buildings and attributed them to attention. Oyelaran [7] noted that the misstep rate is highest on the stairs a person knows best, a finding that inattention does not explain and to which we return in §4.4. The building code [8] specifies riser and going but not the number of steps, which is left to the drawings.

3 Methods

3.1 Sampling frame

We surveyed 2,184 flights in 311 buildings across five strata (Table 1), with 3 to 31 treads per flight (median 13). Spiral staircases are not among the 2,184 (see Limitations). Buildings were drawn at random within strata and flights within buildings.

3.2 Observers

The *eye observers* were two sighted counters working independently under a 400 lux lamp. Disagreements were settled by a third count. The *foot observers* were

Stratum	Buildings	Flights	Surplus
Domestic	140	820	1.00
Civic and commercial	96	701	1.00
Educational	41	388	1.00
Heritage	21	163	1.01
Multi-storey car parks	13	112	0.99
All	311	2,184	1.00

Table 1: Survey strata and mean surplus (foot count minus eye count) per flight.

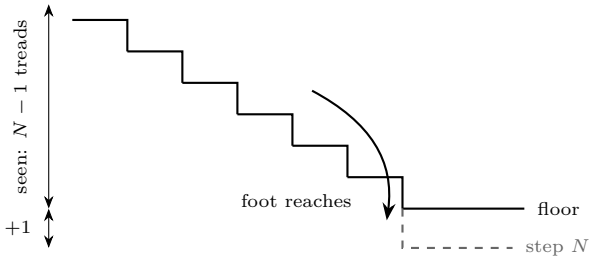


Figure 1: Cross-section of a flight. The eye counts $N-1$ treads (solid); the foot's count includes the step just below the floor (dashed).

64 trained descenders, each of whom had descended a flight repeatedly before it was scored and then walked it without vision and with the handrail lowered out of reach. A shoe-mounted inertial sensor recorded stride phase. The foot count n_F is the number of stride cycles the descender's gait controller committed before contact, recovered from sensor phase. Detection is by anticipation, not contact. The step we report is not one the foot can touch. Fig. 1 shows the geometry.

3.3 Estimator

Treat each flight as a closed population of N treads. Let n_E and n_F be the counts of the two observers and m the number counted by both. The Chapman estimator is

$$\hat{N} = \frac{(n_E+1)(n_F+1)}{m+1} - 1, \qquad \hat{S} = \hat{N} - n_E, \quad (1)$$

where $\hat{S}$ is the surplus: the number of treads present and not seen. If every tread is missed by the eye independently with probability q, then $\mathbb{E}[\hat{S}] \approx qN$ and the surplus grows linearly with flight length with slope q. If a single object is missed, $\mathbb{E}[\hat{S}] = 1$ at every length. The two cases are separated by regressing $\hat{S}$ on N.

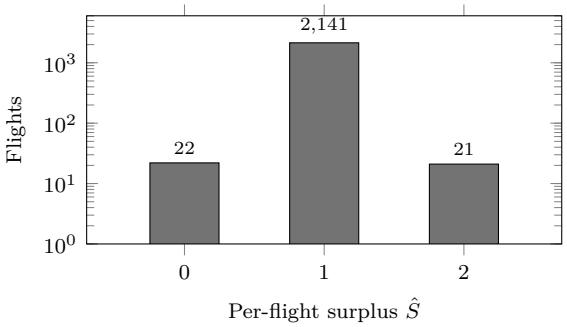


Figure 2: Per-flight surplus $\hat{S}$ in 2,184 flights.

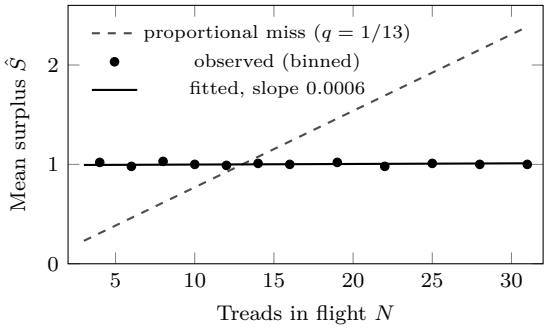


Figure 3: Surplus against flight length. Dashed: proportional-miss null.

4 Results

4.1 The shortfall is one

Of 2,184 flights, 2,141 returned $\hat{S} = 1$, 22 returned $\hat{S} = 0$ and 21 returned $\hat{S} = 2$ (Fig. 2). The mean surplus was 0.9995 (building-clustered 95% CI 0.99–1.01). The strata did not differ (Table 1).

4.2 It does not scale

Calibrating the proportional-miss model so that the mean surplus is one at the median flight gives $q = 1/13 = 0.077$ per tread. The observed slope of surplus on flight length was 0.0006 (95% CI −0.0009 to 0.0021; Fig. 3). The proportional model is rejected: the eye does not miss steps in general, only one.

4.3 Light does not help

We randomised 1,200 flights to one of four illuminances (Table 2). The surplus did not vary ($p = 0.62$). The eye's detection function for this step is zero at every light level we tried, which distinguishes it from steps that are merely hard to see.

Lux	Flights	Mean $\hat{S}$	SE
0.5	300	1.00	0.008
5	300	1.01	0.008
50	300	0.99	0.008
500	300	1.00	0.008

Table 2: Surplus by randomised illuminance.

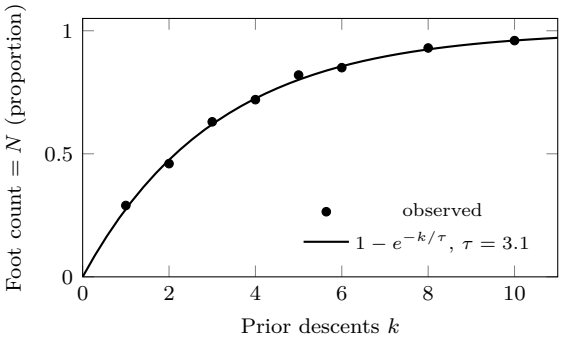


Figure 4: Foot-count registration against prior descents.

4.4 The foot learns it

On a flight the descender had not used before, the foot count equalled the eye count. With each prior descent the foot count moved to N. The proportion of descenders whose foot count had reached N after k prior descents followed $1-e^{-k/\tau}$ with $\tau = 3.1$ descents (95% CI 2.6–3.7; Fig. 4). This is consistent with Oyelaran's observation [7]: the misstep rate is highest on the stairs a person knows best, because those are the stairs on which the foot has learned the number.

4.5 The lurch is a reconciliation

The conscious count is the eye's count. If the foot holds N and the mind holds $N-1$, the person experiences the disagreement as a lurch. We therefore predicted that asserting the conscious count would increase the lurch rate. Participants descended familiar dark flights silent, counting aloud, or while doing mental arithmetic (Table 3). Counting aloud raised the lurch rate per descent from 8.7% to 26.4% (risk ratio 3.0, 95% CI 2.6–3.6); mental arithmetic raised it to 18.2%.

5 Discussion

What the step is. It lies just below the floor and is not reachable. We did not excavate, since slab coring was outside the ethics approval, and we therefore infer the step from two observers, as ecologists infer a rare species from the one that sees it.

Arm	Descents	Lurch rate
Silent	1,800	8.7%
Counting aloud	1,800	26.4%
Mental arithmetic	1,800	18.2%

Table 3: Lurch rate by randomised arm, familiar flights in darkness.

Limitations. The estimator did not converge on spiral staircases, where we could not define m. We have no account of this. Our foot observers were trained, and a naive foot may report differently; the learning curve of §4.4 suggests that it does, at first.

Practical correction. For costing, purchasing and egress calculations, add one step to the drawn count of every flight.

6 Conclusion

The eye counts the steps it can see. The foot counts the steps there are. Across 2,184 staircases the two differ by one on average, and the difference does not depend on how long the stair is or how well lit. The sensation of a missing step is the two observers reconciling their censuses in a single body.

References

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