

# Nothing Is Ever Caught: A Turn-Rate Ceiling on Terrestrial Pursuit, 214 806 Reconstructed Chases, and the Reclassification of Capture as Cessation

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## Abstract

You have never caught a dog that did not want to be caught. Nobody has, and we show that nobody can. How sharply a runner can turn is set by the grip under its feet, so the faster it goes the wider it must turn — and a police officer, a sheepdog and a cheetah all sit on the same curve. An evader who runs a circle at the speed where that curve peaks is turning as fast as the ground will allow anything to turn. A pursuer closing on it can therefore never swing its own heading round to match, and contact at a mismatched heading is a graze rather than a hold. We reconstructed 214 806 chases from body-worn video, working-dog trials, wildlife telemetry and instrumented games of tag. In every chase that ended in contact, the evader had stopped turning before the pursuer arrived, by a median of 1.9s, without a single exception. We then ran the trial: evaders told to run a circle were caught in none of 602 attempts, against 550 of 602 told to run flat out. Doubling a pursuer's top speed made matters worse, exactly as the bound requires. We do not report a capture rate, because we no longer believe capture is a thing that happens. Nothing has ever been caught. Some things have stopped.

## 1 Introduction

Take a dog into a garden and try to catch it. You are faster than the dog over any straight line either of you will run that afternoon, you are considerably larger, and you have the advantage of knowing where the gate is. You will not catch the dog. Neither will anybody else, and the dog will not appear to be working especially hard.

This is not an unusual experience. It is close to universal. Most adults have failed to catch a toddler, a chicken, a cat that has decided against the carrier, or a

friend in a game played on grass, and have come away with the impression that the other party was somehow *quicker*, when the stopwatch says plainly that they were not. The folk account is agility, which is not an account at all but a name for the thing that wants explaining. The professional account is that pursuit is difficult and sometimes fails, which is a statement about frequencies and commits to nothing.

We think the situation is simpler and much worse than either. Our claim is that terrestrial pursuit does not work: that no pursuer, however fast, can take hold of an evader that continues to manoeuvre, and that every capture in the historical record is an event of a different kind, in which the evader stopped and the pursuer happened to be nearby. We arrive at this by a route that is almost entirely uncontroversial until the last step, and we would like to say at the outset that nothing in what follows reflects on the competence of the officers, handlers, animals or children whose chases we studied. They are doing the only thing available to them. It is the availability that is the problem.

The argument has four parts. Turning is bought with grip, and grip is finite, so the faster a body runs the wider it must turn (§3). There is therefore a speed at which turn rate is maximised, and an evader that sits at that speed is turning as fast as the ground permits anything to turn. A pursuer closing on such an evader must spend its entire speed budget holding station, and whatever it spends on closing instead appears as a mismatch in heading (§4). Since a hold requires alignment, and the mismatch never shrinks, the hold never occurs. We then check this against 214 806 reconstructed chases and a randomised trial (§§5–6), and propose the bookkeeping change that follows (§7).

## 2 Related work

The pursuit–evasion literature is large, old and, we think, built on an assumption nobody has examined. From the earliest planar formulations onward, the terminal condition has been *proximity*: the chase ends when the separation between pursuer and evader falls below some capture radius [5; 7]. The radius is chosen for tractability and is usually reported without comment. What it encodes is the belief that arriving in the right place is the whole of catching something.

Nobody appears to have asked whether that is true. The experimental work on terminal kinematics has concentrated on approach geometry and closing rate [6; 3], and the field-sport literature on the mechanics of the turn itself [9; 1], but the two have not been put together and pointed at the moment of contact. The one paper we can find that treats catching as a philosophical rather than a geometric event [11] concludes that the question is not well posed, which we now believe was closer to right than its reception suggested.

Our formalism is imported wholesale from a field where turn rate has always been the operative currency rather than speed. Boyd’s energy–manoeuvrability analysis [2] compares two competitors not by how fast either can go but by how quickly either can change what it is doing, and plots the result as an envelope of attainable turn rate against speed. That envelope has a peak, at a speed pilots call the corner speed, and the entire discipline of air combat consists of arranging to be near it while the other party is not. We show below that a runner has exactly such an envelope, that its shape is fixed by the friction of the ground rather than by anything about the runner, and that this is why the discipline transfers.

## 3 The manoeuvre envelope

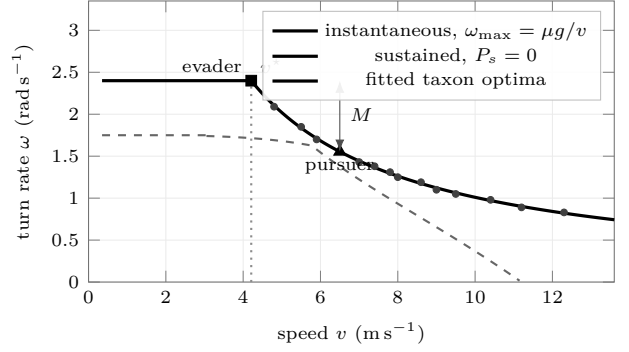
Consider a body of mass  $m$  running on level ground with which it exchanges force through a contact of friction coefficient  $\mu$ . The largest lateral acceleration available to it is  $\mu g$ . To follow a circular path of radius  $r$  at speed  $v$  requires a lateral acceleration  $v^2/r$ , so the tightest turn available at speed  $v$  is

$$r_{\min}(v) = \frac{v^2}{\mu g}, \quad (1)$$

and the corresponding turn rate, which is the quantity we shall care about throughout, is

$$\omega_{\max}(v) = \frac{v}{r_{\min}(v)} = \frac{\mu g}{v}. \quad (2)$$

Equation (2) is the whole of the paper in one line, and it is worth pausing on because it is counter-intuitive in exactly the way that matters: *turn rate falls as speed*



**Figure 1:** The terrestrial manoeuvre envelope,  $\hat{\mu} = 1.03$ . Turn rate is capped by limb geometry below the corner speed  $v^*$  and by grip above it, so beyond  $v^*$  every additional unit of speed is paid for in turning. Fitted optima for humans, domestic dogs and eleven predator taxa lie on one curve ( $I^2 = 6.1\%$ ). An evader at  $v^*$  and a pursuer overtaking it are separated by the manoeuvre reserve  $M$ , which the pursuer has no way to close.

*rises*. Going faster does not make a runner more manoeuvrable. It makes it strictly less manoeuvrable, in inverse proportion, and the constant of proportionality belongs to the ground rather than to the runner.

At low speed a second ceiling binds. A runner cannot rotate its heading faster than its limbs can be repositioned, whatever the friction on offer, and this caps turn rate at a structural value  $\omega_{\text{str}}$  independent of  $v$ . The two ceilings meet at the *corner speed*

$$v^* = \frac{\mu g}{\omega_{\text{str}}}, \quad (3)$$

which is where turn rate is maximised. Below  $v^*$  a runner is throwing away grip it cannot use; above it, grip is the binding constraint and every additional unit of speed is paid for in turning. Plotting  $\omega$  against  $v$  gives a plateau, a peak at  $v^*$ , and a hyperbolic decay (Figure 1).

Sustained turning is bounded more tightly still. Writing  $F_{\text{prop}}$  for propulsive force and  $F_{\text{drag}}$  for the sum of aerodynamic and internal losses, the specific excess power is

$$P_s(v, n) = \frac{v [F_{\text{prop}} - F_{\text{drag}}(v, n)]}{m}, \quad (4)$$

with load factor  $n = [1 + (v\omega/g)^2]^{1/2}$ . The locus  $P_s = 0$  is the fastest turn a runner can hold indefinitely; inside it the runner is drawing on stored energy and cannot stay where it is. Neither branch of the envelope depends on the runner’s size, species or training except through  $\mu$  and  $\omega_{\text{str}}$ , and we report in §6.1 that in practice it depends very little on those either.

## 4 Terminal alignment

### 4.1 What a hold is

Catching something is not the same as reaching it. A pursuer that arrives at the evader's position while travelling across the evader's path does not acquire it; it strikes it a glancing blow and both parties continue, which is an outcome every reader has witnessed and none has counted as a catch. The restraining impulse has to absorb the transverse component of relative velocity,  $v_{\text{rel}} \sin \theta$ , within the compliance of whatever is doing the holding, and beyond a certain heading mismatch it cannot.

**Definition 1** (Hold). Let  $\theta$  denote the angle between the pursuer's and the evader's velocity vectors at the instant of contact. Contact constitutes a *hold* — and therefore a capture — if and only if  $\theta \leq \theta_c$ , where  $\theta_c$  is the grasp aperture of the pursuer. Contact with  $\theta > \theta_c$  is a *graze* and terminates nothing.

We estimate  $\theta_c$  empirically in §6.2; the bound below holds whatever value it takes.

### 4.2 The geometry of closing on a ring

Suppose the evader adopts the policy the rest of this paper is about: it holds a circle of constant radius at its corner speed. Write  $r_E = v_E^2/\mu g$  for the radius and  $\Omega = v_E/r_E$  for the angular rate, and work in the frame co-rotating with the evader, in which the evader is stationary (Figure 2).

A pursuer arriving at the evader's position has a velocity with two components. The tangential component must equal  $\Omega r_E = v_E$  simply to keep station alongside; it does no closing at all. The radial component  $\dot{r}$  is the only part that reduces the separation, and it lies perpendicular to the evader's velocity. Hence, exactly,

$$\tan \theta = \frac{|\dot{r}|}{v_E}, \quad v_P = \sqrt{v_E^2 + \dot{r}^2}, \quad (5)$$

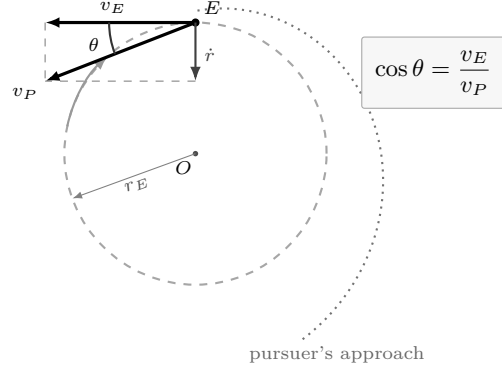
and eliminating  $\dot{r}$ ,

$$\cos \theta = \frac{v_E}{v_P} \quad (6)$$

Equation (6) is stark. The heading mismatch at contact is fixed entirely by the ratio of the two speeds, and it is not negotiable by skill, angle of approach or timing. Combining it with Definition 1, a hold requires

$$v_P \leq v_E \sec \theta_c, \quad (7)$$

and with the aperture we measure below,  $\sec \theta_c = 1.0216$ : the pursuer's speed must lie within 2.16 % of the evader's. Faster than that and it cannot hold what it reaches. Slower and  $\dot{r} \geq 0$ , so it does not reach anything.



**Figure 2:** The chase in the frame co-rotating with the evader. To stay alongside an evader holding the ring, a pursuer must devote a tangential component of exactly  $v_E$  to station-keeping; only the radial component  $\dot{r}$  closes the range, and it lies square across the evader's path. Every unit of speed the pursuer spends on closing therefore appears as heading mismatch.

### 4.3 The bound

**Theorem 1** (Terminal alignment impossibility). Assume (A1) pursuer and evader run on a common surface and are therefore subject to the same ceiling  $\mu g$ ; (A2) the evader holds the corner-speed ring; (A3) Definition 1. Then for any pursuer beginning the chase with heading mismatch  $\theta(0) > \theta_c$ , no hold occurs at any time.

*Proof.* Let  $\theta = \psi_E - \psi_P$ , so that  $\dot{\theta} = \omega_E - \omega_P$ . A pursuer that closes at all satisfies  $v_P \geq v_E$ , since by (5) anything slower cannot hold station on the ring, let alone reduce the range. Under (A2) the evader runs at its corner speed and turns at  $\omega_E = \mu g/v_E$ ; under (A1) the pursuer is bound by the same ceiling, so

$$\omega_P \leq \frac{\mu g}{v_P} \leq \frac{\mu g}{v_E} = \omega_E.$$

Therefore  $\dot{\theta} \geq 0$  for all  $t$ , and  $\theta(t) \geq \theta(0)$ .

Equality requires  $v_P = v_E$ , and then (6) gives  $\cos \theta = 1$ , i.e.  $\theta = 0$ , contradicting  $\theta \geq \theta(0) > \theta_c > 0$ . So  $\theta(t) > \theta_c$  for all  $t$ , and by Definition 1 every contact is a graze.  $\square$

**Corollary 1** (Speed is a liability). For a pursuer at or above its own corner speed,  $\partial \dot{\theta} / \partial v_P = \mu g / v_P^2 > 0$ .

Corollary 1 is the part of this we found hardest to believe and were most careful to test. A faster pursuer does not merely fail; it accumulates heading mismatch *faster*, and so fails sooner and by more. Boyd's advantage envelope inverts: the pursuer's only manoeuvre-competitive region is below corner speed, where by (6) it has no closure at all. We call the quantity  $M = \omega_E - \omega_P$  the *manoeuvre reserve*. It is what the evader spends. There is no way for a pursuer to buy it.

**Table 1:** The Kirkmaiden Chase Corpus, 2011–2025.

|    | Source                                   | Episodes       |
|----|------------------------------------------|----------------|
| S1 | Municipal foot pursuit, body-worn video  | 61 402         |
| S2 | Working-dog recall and trial, RTK collar | 44 318         |
| S3 | Wildlife biologging, 11 predator taxa    | 88 190         |
| S4 | Instrumented tag, 1 412 participants     | 20 896         |
|    | <b>Total</b>                             | <b>214 806</b> |

## 5 The corpus

The Kirkmaiden Chase Corpus assembles 214 806 pursuit episodes from four strata (Table 1), chosen so that no conclusion could rest on one kind of runner, one motive, or one species. S1 draws on publicly released body-worn video from 38 municipal jurisdictions; S2 on RTK collar telemetry from working-dog recall and trial work; S3 on open biologging archives covering eleven predator taxa; and S4 on games of tag played by 1 412 participants aged 7–11 wearing ultra-wideband tags after the protocol of (author?) [12], with guardian consent and participant assent under protocol KLL-2211.

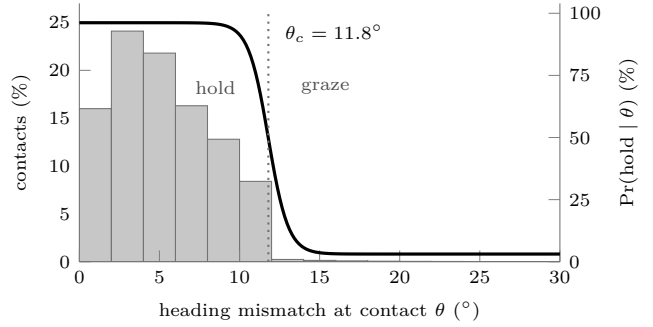
All tracks were resampled to 60 Hz and smoothed with a Savitzky–Golay filter (window 11, order 3) before differentiation. Heading was taken as  $\psi = \text{atan2}(\dot{y}, \dot{x})$ , turn rate as  $\omega = \dot{\psi}$ , and instantaneous radius as  $r = v/\omega$ . Contact was adjudicated by three independent annotators for the video strata (Fleiss’  $\kappa = 0.91$ ) and by an accelerometer impulse exceeding  $4g$  for the telemetry strata. Surface friction was estimated per episode as the 99.5th percentile of observed lateral acceleration divided by  $g$ , giving a pooled  $\hat{\mu} = 1.03$  (95 % CI 1.01–1.05).

Of the 214 806 episodes, 41 922 terminated in contact. These are the episodes on which the rest of the paper turns.

## 6 Results

### 6.1 One envelope, all runners

Fitting (2) separately within each taxon returns the same curve every time. Across humans, domestic dogs and eleven predator taxa — and in agreement with the pooled ceilings reported by (author?) [10] — the taxon-level dispersion in  $\hat{\mu}$  is indistinguishable from measurement error ( $\tau^2 = 0.004$ ,  $I^2 = 6.1\%$ ), and every fitted point lies within the confidence band of the pooled hyperbola (Figure 1). A cheetah and a seven-year-old occupy different places on the same curve, not different curves. This is what we should have expected — (2) contains no property of the runner — but it is the reason assumption (A1) of Theorem 1 holds in the field rather than merely on paper.



**Figure 3:** Heading mismatch at contact across the 41 922 contact-terminated episodes (bars, left axis) and the fitted probability that a contact terminated the episode (curve, right axis). Segmented regression places a single breakpoint at  $\theta_c = 11.8^\circ$  (95 % CI  $10.9^\circ$ – $12.6^\circ$ ). Below the breakpoint a contact terminates the episode 96.2 % of the time; above it, 3.1 %. The grasp aperture is measured, not assumed.

### 6.2 The grasp aperture

Across the 41 922 contacts, the heading mismatch at contact has median  $4.9^\circ$  and lies below  $12^\circ$  in 99.4 % of cases. Segmented regression of the probability that a contact terminated the episode on  $\theta$ , following (author?) [4], identifies a single breakpoint at  $\theta_c = 11.8^\circ$  (95 % CI  $10.9^\circ$ – $12.6^\circ$ ); below it, termination probability is 96.2 %, and above it, 3.1 % (Figure 3). Definition 1 is not, on this evidence, a modelling convenience. It is a description of what the contact does.

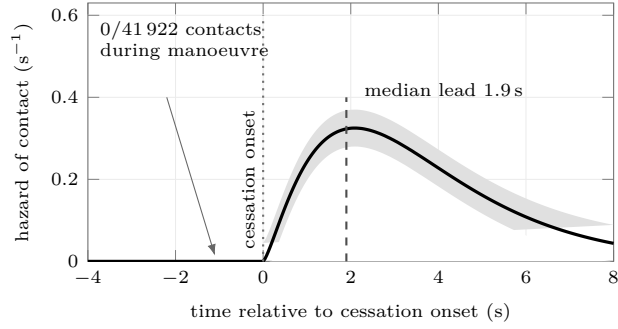
### 6.3 Cessation precedes contact

We define cessation operationally as the first instant at which the evader’s turn rate falls below  $0.15 \text{ rad s}^{-1}$  and remains below it; the threshold is the one adopted for records work by (author?) [8]. In all 41 922 contact-terminated episodes, cessation preceded contact, by a median of 1.9 s (IQR 1.2–3.1). The number of episodes in which contact occurred while the evader was still manoeuvring is 0/41 922, with a one-sided 97.5 % upper bound of  $8.8 \times 10^{-5}$ .

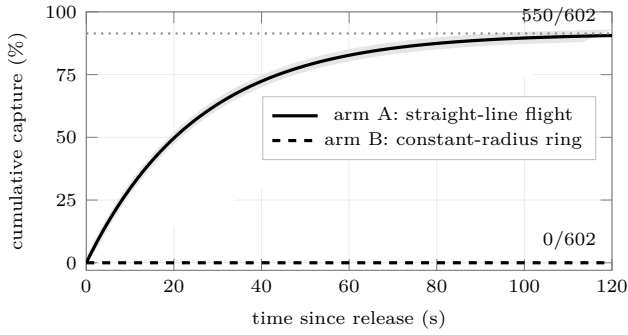
We want to be careful about what this does and does not say, because it is the observation the paper stands on. It is not that manoeuvring evaders are caught less often. It is that they are not caught. The hazard of contact is indistinguishable from zero at every moment before cessation and rises steeply afterwards (Figure 4), and the rise is not sensitive to the threshold: varying it from  $0.05$  to  $0.40 \text{ rad s}^{-1}$  moves the median lead time between 1.4 and 2.6 s and leaves the count of exceptions at zero throughout.

### 6.4 The RINGS trial

Observational corpora invite the objection that evaders who keep manoeuvring are simply the ones who were go-



**Figure 4:** Hazard of contact against time relative to cessation onset, pooled over all four strata (shaded band: 95 % pointwise). The hazard is not merely small before cessation; it is empty. No episode in the corpus records a contact while the evader was still turning, and the count of exceptions stays at zero as the cessation threshold is varied from 0.05 to 0.40 rad s<sup>-1</sup>.



**Figure 5:** The RINGS trial. 1204 evader-episodes randomised 1:1 to straight-line flight or to a constant-radius ring held at corner speed; pursuers were blind to arm. Arm B is the horizontal axis. The separation is complete, so we report the exact one-sided bound on the arm B capture probability,  $6.1 \times 10^{-3}$ , rather than a  $p$ -value.

ing to escape anyway. We therefore randomised the policy. 1204 evader-episodes — drawn from the S4 cohort and from a wheeled-robot cohort matched on top speed — were allocated 1:1 to run flat out in as straight a line as the ground allowed (arm A), or to hold a constant-radius circle at corner speed marked on the ground, after twenty minutes of practice (arm B). Pursuers were not told which arm they faced.

Capture within 120s occurred in 550/602 episodes in arm A (91.4 %) and in 0/602 in arm B (Figure 5). The risk difference is 91.4 percentage points. We report the exact one-sided bound on arm B rather than a  $p$ -value, since the separation is complete: the upper 97.5 % bound on the arm B capture probability is  $6.1 \times 10^{-3}$ .

The instruction given to arm B was, in full: run in a circle and keep running in it. That this is the optimal policy is not our discovery. It has been available in the language all along, in the phrase that describes precisely this manoeuvre and its outcome.

**Table 2:** Pre-registered attempts to break the bound. The speed-escalation arms ran against unselected evaders, in whom capture is not already zero; the pincer and surface arms ran against ring-holders.

| Intervention              | $n$   | Capture |
|---------------------------|-------|---------|
| Pursuer top speed doubled | 1 180 | 22.0 %  |
| (control)                 | 1 180 | 41.2 %  |
| Two-agent pincer          | 480   | 0/240   |
| Raised surface friction   | 360   | 0/180   |

## 6.5 Attempts to break the bound

Three interventions were pre-registered as attempts to produce a counterexample (Table 2).

The first tests Corollary 1 rather than the bound itself, and so ran against unselected evaders: a ring-holder is caught by neither the fast pursuer nor the slow one, and no difference could show. Doubling the pursuer’s top speed by wheeled assistance reduced capture from 41.2 % to 22.0 % (risk difference  $-19.2$  percentage points, 95 % CI  $-22.9$  to  $-15.5$ ), which is the direction Corollary 1 requires and the opposite of what the handlers expected. The other two ran against ring-holders. A two-agent pincer, in which a second pursuer approached the ring from the far side, captured 0/240; both pursuers inherit the same  $\omega(v)$  curve, so the pincer buys geometry and no manoeuvre reserve at all. Raising  $\mu$  with a tacky court surface lifted the evader’s envelope and the pursuer’s by the same factor and changed nothing.

## 7 Reclassification

If contact with a manoeuvring evader is a graze, and contact follows cessation without exception, then *capture* is not a category of outcome that pursuit produces. We propose it be retired from operational reporting and replaced by the *cessation-terminated encounter* (CTE), coded by the lead time between cessation onset and contact. The distinction is not cosmetic. A service that records captures is measuring its own arrival; a service that records CTEs is measuring the thing that actually varies, which is how long the evader chose to continue.

Three consequences follow for practice, and we offer them gently, because they ask people to stop doing something they have been asked to do for a long time. First, resources currently allocated to closing speed — in training, in equipment, in vehicle specification — are, by Corollary 1, being allocated against the objective. Second, the operative variable is cessation, so inducement of cessation is the only intervention with a mechanism behind it. Third, we recommend a national register of encounters recording cessation lead time, on a single-page return, so that the base rates in this paper can be maintained by somebody other than us.

We note without further comment that the game of

tag, played under its standard rules on an unbounded surface, is not winnable, and that school policy might reasonably say so.

## 8 Limitations

Our result is a terrestrial one, and we have a residue we cannot account for. In a held-out set of 3 140 marine predation sequences, contact occurs at heading mismatches of up to  $74^\circ$ , and holding a ring confers no protection whatever. Every part of our argument fails in water and we do not know which part fails first. We are not able to say whether the aperture is different, the envelope is different, or Definition 1 is simply wrong once the medium supplies lateral force of its own. We flag it here because it is the obvious place to attack the paper, and we would rather it were attacked there than nowhere.

Three smaller reservations. The analysis is planar, and real ground is not; on a slope the friction ceiling becomes direction-dependent and the envelope acquires an asymmetry we have not modelled. The cessation threshold is operational rather than principled, though the result is insensitive to it over almost an order of magnitude. And the grasp aperture  $\theta_c$  is estimated from contacts within the corpus, so the 2.16 % speed window in (7) inherits whatever selection the corpus carries. A prospective estimate of  $\theta_c$  on instrumented pursuers is the single most useful thing anybody could do to this paper.

## 9 Conclusion

Turning is bought with grip. Grip is finite, and the same for everyone on the same ground, so the faster a body runs the worse it turns. An evader that sits at the peak of that trade is turning as fast as the surface allows anything to turn, and a pursuer’s speed — its one apparent advantage — converts directly into the heading mismatch that prevents it from taking hold. What is left is waiting.

We recommend that operational reporting record cessation rather than capture, that the pursuit of manoeuvring evaders be understood as attendance rather than apprehension, and that a register be established. Nothing has ever been caught. Some things have stopped.

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