Excavation by Avoidance: The Footpath Puddle Is Dug by the People Who Walk Around It

An Eleven-Year Instrumented Record of Four Unsealed Routes, a Painted-Disc Trial, and a Randomised Remediation Protocol

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Received 12 May 2026; accepted 9 August 2026

Abstract

There is a puddle on the path to the west gate. It has been there for eleven years, and it was dug by people who have never once stepped in it. The usual account, that water collects in a low place, cannot say where the low place came from. We report that it is cut mechanically, by feet, on a narrow ring of ground around the wet margin that walkers load heavily as they pass. Ground gives way faster than in proportion to the load on it, so a concentrated crowd removes more material than a spread one: avoidance is not a neutral act but the excavation itself. Across four unsealed footpaths and 1,908,442 instrumented passages, rim lowering tracks how tightly footfall is concentrated (exponent 1.83, 95% CI 1.71–1.95) and does not track rainfall at all. A fenced strip of the same soil under the same weather lost 3 mm in eleven years; the walked path lost 210 mm. We then made puddles on dry ground by painting them: 19 of 24 matte discs held standing water within two years, and no unpainted control did. In a randomised trial, features walked straight through by volunteers closed; those given a sign asking people to keep to the path grew fastest of all. There is exactly one way to remove a puddle: walk into it.

Keywords: footpath erosion; avoidance behaviour; trail widening; surface processes; path remediation

1 Introduction

The path to the west gate is 240 m long and unsealed. At a point 71 m from the road there is a puddle. It is there now; it was there when this survey began in March 2015; and the earliest photograph of it we have been able to find, taken for an unrelated drainage report, is dated 1998. It is 2.9 m across. In 2015 it was 1.4 m across.

Nobody in the grounds department had ever thought this required an explanation, and for the first four years neither did we. Water collects in a low place. The difficulty with that sentence is that it is not an explanation of anything. It is a restatement with the causal arrow left blank, and when the arrow is supplied it points backwards: the low place, one is told, was made by the water. Standing water on a footpath does not flow, carries no bedload, and exerts a shear stress on the order of 1×10^{-3} Pa, some five orders of magnitude below the yield threshold of the compacted hoggin it sits on [4]. It cannot dig.

This paper reports that the depression is cut by feet, and specifically by the feet of people who are taking care not to step in it. The mechanism is, in retrospect, visible from any window overlooking a wet path. Walkers approaching standing water do not step to its edge; they leave a clearance, and they place a full-weight stride at the outer limit of it. The consequence is that the ground immediately outside the wet margin receives the entire traffic of the path across a strip roughly a third of its width. Because compacted soils lose strength under repeated loading faster than in proportion to the load [4, 9], the concentration is not a redistribution of damage but a multiplication of it. The ring subsides. The wet margin advances into the subsided ring. The walkers re-measure their clearance from the new margin and begin again, a little further out.

English already has the measurement. To *give a wide berth* is, in the original nautical sense, to leave a stated clearance between a hull and a hazard, and eighteenth- and nineteenth-century practice specified it in fathoms according to the hazard's class [8]. Ashore the phrase survives only as a figure of speech for prudence. We use it here in the older sense. The clearance a walker leaves between sole and wet margin is the most informative variable in the system, and there is no reason to coin a term for a quantity that already has one.

Contributions. (i) We define the berth b and the clearance ratio $c = b/r$ and show the latter is stable at $c = 0.19$ across four soils and a forty-fold range of traffic. (ii) We prove that, for a fixed number of walkers, concentrating footfall strictly increases the ground removed, and put the penalty for avoidance at 2.7× (Proposition 1). (iii) We derive a critical traversal fraction $\phi^* = 0.34$ separating unbounded growth from closure (Proposition 2). (iv) We report eleven years of instrumented profilometry in which rim lowering tracks footfall concentration and not rainfall, together with a rainless interval and a fenced placebo path that between them exclude water as necessary or sufficient. (v) We manufacture puddles on dry ground by painting them. (vi) We report a randomised trial in which direct traversal closes features and signage accelerates them.

2 Related work

The widening of unsealed routes under recreational traffic is well described. Bewick and Sturrock [2] audited 311 permissive paths over five years and found mean width increasing at 34 mm per year, with the increase concentrated at wet points; Sanderling [10] formalised the same observation as braiding, a load-shedding response in which a route splits around an obstruction. Both read the wet feature as the cause and the widening as the response. The observation is correct in every particular. The reading of it is inverted.

The soil-mechanical ingredient is older. Halloran [4] established shear-excess detachment in compacted loams, with lowering rate rising as a power of load rate above a reconsolidation threshold; the exponent is attributed to cyclic degradation of undrained strength, whereby a surface reloaded before pore pressures dissipate presents a weaker face to each successive pass. Ratcliffe [9] showed that any such convex response makes aggregate damage a strictly convex functional of the load distribution, though his interest was in vehicle tracking and he did not consider pedestrians at all. Proposition 1 is his result specialised to a distribution the pedestrian chooses.

On intervention, Vardy and Colquhoun [11] meta-analysed 88 on-path behavioural signs and reported a pooled compliance effect of +0.31 standard deviations in stated intention with no detectable effect on measured route condition; the authors describe the gap as a puzzle. Okonkwo [7] treats desire lines as revealed preference over walking cost; we adopt that reading in §3 to interpret the berth. Nyberg and Alonzo [6] gives the freeze–thaw contribution we control for, and Ilves [5] the estimator for slow surface change from repeated cross-sections. Finally Croom [3] documents depression formation on sealed asphalt footways at rates the surface's yield threshold appears to forbid; we return to this in §6.1, having failed to account for it.

3 The berth model

3.1 Definitions

Let a feature be circular of radius r, centred on a tread of usable width W. For passage i, let x_i be the lateral offset of the near edge of the loaded sole from the feature centreline.

Definition 1 (Berth)**.** The *berth* b is $m - r$, where m is the mode of $|x_i|$ taken over avoiding passages, those with $|x_i| > r$: the distance in metres between the wet margin and the nearest habitually loaded ground.

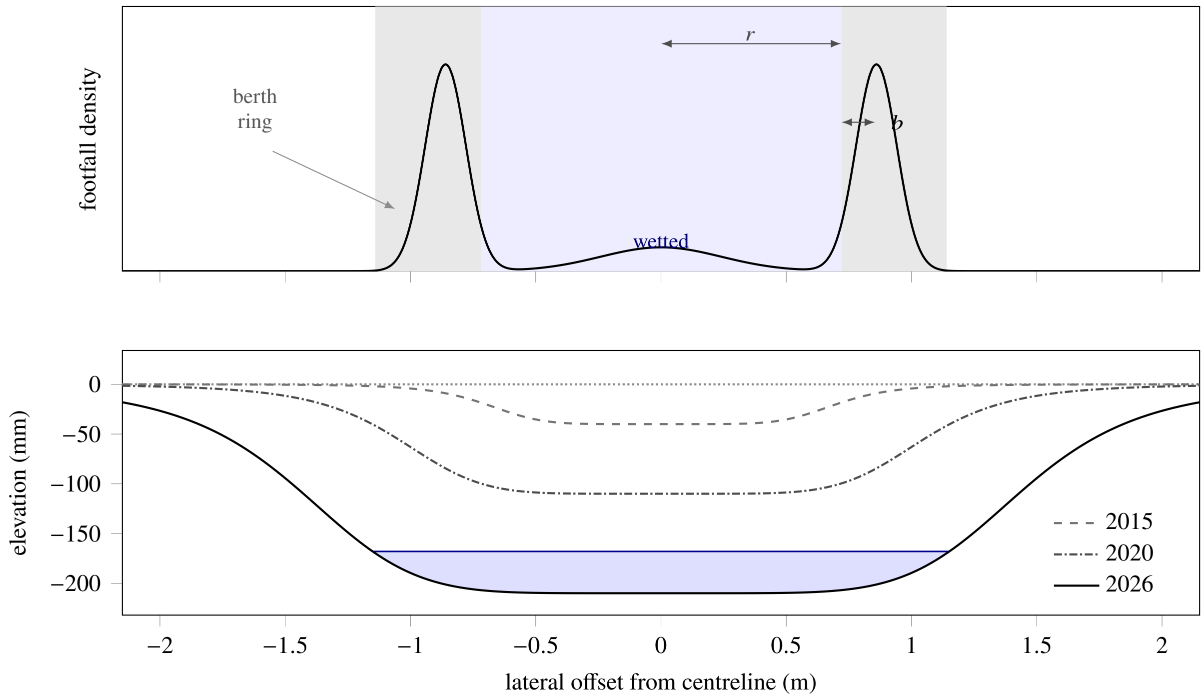

Figure 1: The berth ring and its record. Load withheld from the centre is not withheld from the path; it is moved to the rim.

Definition 2 (Clearance ratio, traversal fraction)**.** The *clearance ratio* is $c = b/r$. The *traversal fraction* is $\phi = \Pr[|x_i| < r]$: the share of passages that go straight through.

Measured offset distributions are sharply bimodal (Figure 1), with modes at $\pm(r + b)$ and a suppressed centre. Across all four sites, pooling 1,908,442 passages, $c = 0.19$ (95% CI 0.18–0.20), with no detectable dependence on r, on soil, on tread width, or on traffic volume. That a ratio should be stable across a forty-fold range of traffic is the first indication that it is a property of the walker rather than of the ground. Read as revealed preference [7], a constant c is what a walker produces when the cost of a wet sole is fixed and the cost of the detour is proportional to the size of the thing being avoided.

3.2 Load and detachment

Write $L(x)$ for the footfall rate per unit width at offset x, so that $\int L = \Lambda$, the daily passage count. Following Halloran [4],

$$\dot{h}(x) \;=\; -\,\alpha\big(L(x) - L_c\big)_+^n, \qquad n > 1, \tag{1}$$

with α a soil constant and L_c the rate below which the surface reconsolidates between passages. L_c is finite only where the peak pressure of a single footfall exceeds the surface's yield stress; where it does not, no traffic rate detaches anything. Our fitted exponent is $n = 1.83$ (95% CI 1.71–1.95); Halloran reports 1.7–2.1 for comparable loams under plate loading, which we regard as agreement.

3.3 Avoidance maximises excavation

Proposition 1 (Concentration penalty)**.** *Let total excavation rate be* $D[L] = \alpha \int (L(x) - L_c)_+^n \, dx$ *with* $n > 1$*, subject to* $\int L = \Lambda$ *and* L *supported on a set of measure* S*. Then*

$$D[L] \;\geq\; \alpha\, S \left(\frac{\Lambda}{S} - L_c\right)_+^n,$$

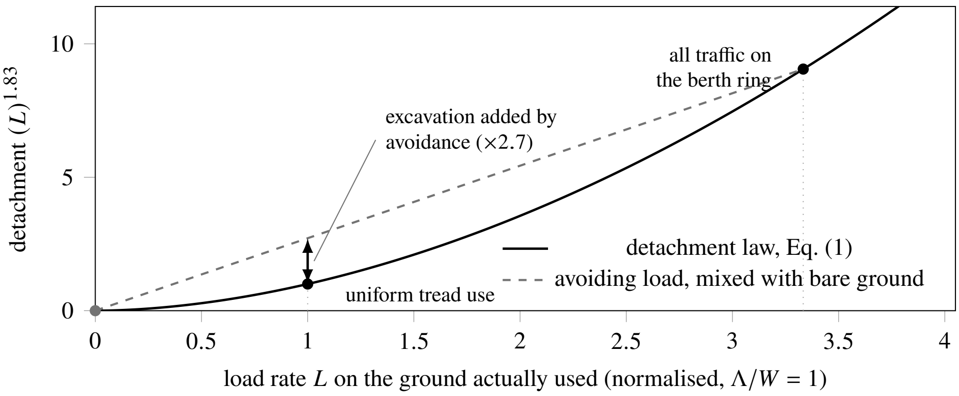

Figure 2: The concentration penalty. Splitting a load between a heavily used strip and unused ground lifts the mean response onto the chord; the vertical gap is the excavation attributable to avoidance.

with equality if and only if L is a.e. constant on its support, or lies a.e. below L_c. Consequently, among load distributions carrying the same traffic, excavation is minimised by uniform tread use and increases strictly under any concentration that leaves the mean above L_c; restricting the support from W to S multiplies excavation by $(W/S)^{n-1}$ in the limit $L_c \to 0$.

Proof. The map $u \mapsto (u - L_c)_+^n$ is convex for $n \geq 1$ and strictly convex on $u > L_c$. Applying Jensen's inequality to the normalised measure dx/S on the support gives $\frac{1}{S}\int (L - L_c)_+^n dx \geq (\frac{1}{S}\int L\,dx - L_c)_+^n$; multiplying by αS gives the bound. Equality requires L to be a.e. constant where it exceeds L_c, and admits the degenerate case $L \leq L_c$ a.e., in which both sides vanish. For the ratio, set $L_c = 0$ and evaluate the uniform case on supports of measure W and S: $\alpha\Lambda^n S^{1-n}$ against $\alpha\Lambda^n W^{1-n}$. □

The specialisation is immediate. Avoidance confines the load to two strips, one either side of the feature, each a loaded sole wide (0.10 m) and broadened by the between-passage scatter in offset (SD 0.04 m): $S = 0.36$ m against a tread of $W = 1.2$ m. S is a property of feet and does not vary with r. With $n = 1.83$,

$$\frac{D_{\text{avoiding}}}{D_{\text{uniform}}} = \left(\frac{1.2}{0.36}\right)^{0.83} = 2.72. \tag{2}$$

The same crowd, taking care, removes 2.7 times as much ground as it would not taking care. Figure 2 gives the geometry of the inequality.

3.4 Growth and the critical traversal fraction

Rim lowering advances the wet margin outward into freshly subsided ground. Direct traversal does the opposite: material sheared from the rim is carried on soles into the depression and consolidated there, a transport term also present in the braiding literature [10]. Absorbing Λ, S and the soil constants into two fitted coefficients, neither of which depends on r, the reduced form is

$$\frac{dr}{dt} = A\,(1 - \phi)^n - B\,\phi. \tag{3}$$

Proposition 2 (Critical traversal fraction)**.** *Equation* (3) *admits no equilibrium in r except in the degenerate case $\phi = \phi^*$, where every r is one. There is a unique $\phi^* \in (0, 1)$ solving $A(1 - \phi^*)^n = B\phi^*$ such that for $\phi < \phi^*$ a feature grows without bound from any initial size, including zero, and for $\phi > \phi^*$ every feature closes in finite time.*

Proof. The right-hand side is independent of r, so $\text{sign}(dr/dt)$ is constant along any trajectory and r is monotone. $A(1 - \phi)^n$ is strictly decreasing in ϕ from A to 0; $B\phi$ is strictly increasing from 0 to B; the two cross exactly once. Closure in finite time follows from $|dr/dt|$ being bounded below by a positive constant for fixed $\phi > \phi^*$. □

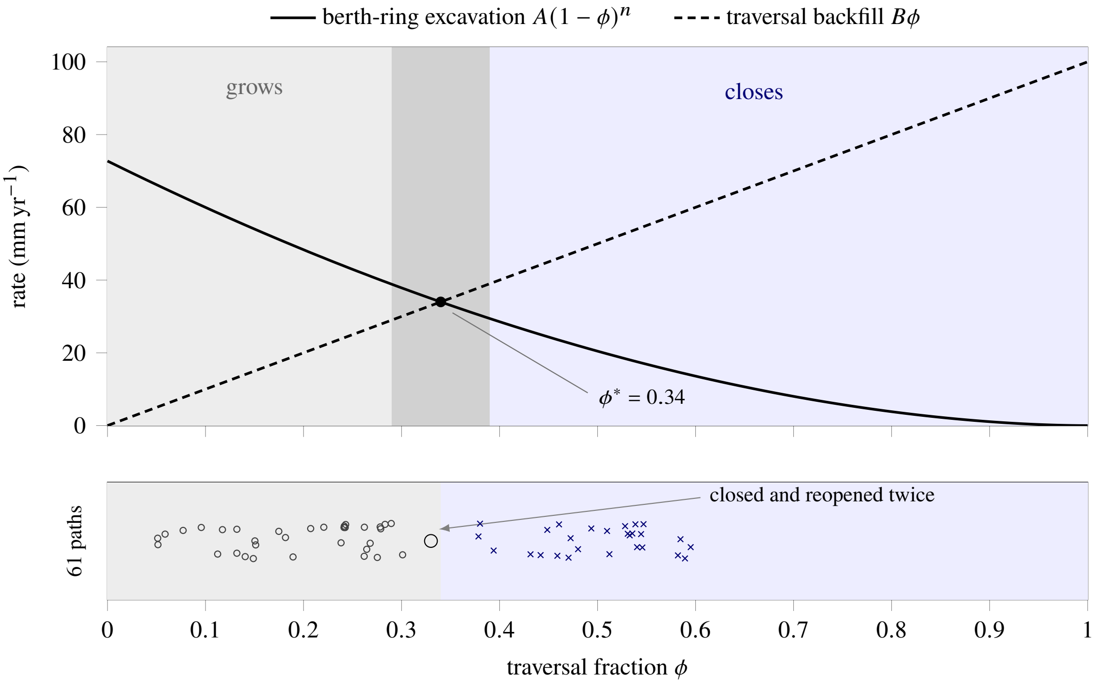

Figure 3: The critical traversal fraction. Roughly one walker in three, straight through, is the whole of the remedy.

Fitting A, B and n jointly to the Site A–D series gives $\phi^* = 0.34$ (bootstrap 95% CI 0.29–0.39; 10,000 resamples clustered on site). Figure 3 plots the two branches. The absence of an interior equilibrium is the model's least comfortable prediction: a path is either accumulating a puddle or it is not, and there is no such thing as a puddle at its natural size.

3.5 Corollaries

Corollary 1 (Courtesy)**.** *dr/dt is convex in ϕ: $\partial^2(dr/dt)/\partial\phi^2 = n(n-1)A(1-\phi)^{n-2} > 0$ for $n > 1$. Growth is therefore superlinear in avoidance: on a route used by more deferential walkers, each further increment of deference costs more ground than the one before it.*

Corollary 2 (Signage is contraindicated)**.** *dr/dt is strictly decreasing in ϕ, and the radius gained per cycle of the rim–margin feedback is increasing in b. A request to keep to the path, to avoid wet ground, or to protect the surface acts on both variables in the damaging direction.*

Corollary 3 (Widening does not help)**.** *The equilibrium condition in Proposition 2 does not contain W. Widening a tread raises the attainable berth and relocates the ring onto virgin ground without altering ϕ^*.*

4 Sites and instrumentation

Five sites were maintained (Table 1). Site E is fenced, unwalked, and lies 40 m from Site A on the same soil under the same weather. It exists to be a placebo path, and the falsification argument of §5.2 rests on it.

Profilometry. Twenty-one fixed stations per site, referenced to benchmark pins driven to 900 mm. Fortnightly 1.2 m cross-sections at 2 mm vertical resolution, by contour gauge to April 2018 and by laser line profiler thereafter [1]. 26,914 cross-sections in total, against a nominal 27,300; the shortfall is missed rounds. The instrument change is a discontinuity we have not removed; it steps against the old gauge to 1 mm on dry stations and rather worse on wet ones, and all models carry an instrument indicator.

Table 1: Survey sites. Passage counts are means over the record.

Site	Description	Substrate	Passages d^{-1}	Record
A	Ashcombe Meadows permissive path	clay loam	410	2015–2026
B	Halewood Cutting, disused railway	cinder over clay	1,240	2017–2026
C	West Gate approach, Ashcombe	compacted hoggin	3,700	2015–2026
D	Barrow Lane bridleway	silt loam	90	2018–2026
E	*Placebo path* — fenced, unwalked	clay loam	0	2015–2026

Extracts from the site notebook, Site A.

2015-03-02. Pins driven. Station 9 sits in the puddle. Will have to profile it wet.
2016-11-14. Station 9 unchanged since spring. Stations 7 and 11 down 4 mm each. Recorded, and did not think about it.
2018-04-30. New profiler. Steps against the old gauge to 1 mm dry, worse wet. Noting the discontinuity rather than correcting it.
2019-03-11. Rim pin 14 exposed 22 mm. Nobody walks there. Nobody walks *there*. Four years to see it.
2022-07-26. No rain since 3 May. Feature dry, floor cracked, still deepening. No explanation for this yet. Writing it down before we lose it.
2024-02-08. Two discs painted over by contractors. Excluded.

Figure 4: Contemporaneous entries, Site A. The record is reproduced as written, including the four years in which nothing was made of it.

Gait gates. Downward-looking infrared beam-break arrays, 32 beams at 40 mm pitch across a 1.28 m corridor, resolving lateral foot placement to ±20 mm. The arrays record beam-break timings only; no imagery is captured at any point.

Plate arrays. Twenty-four buried piezoresistive cells (0.1 MPa to 1.2 MPa) at 60 mm pitch, giving peak pressure per footfall by offset, calibrated quarterly against a 70 kg drop mass.

Wetness. Capacitance probes at 30 mm, 80 mm and 150 mm depth; a tipping-bucket gauge; and a fixed-mast photograph of wetted extent at 07:00 daily.

Figure 4 reproduces the Site A notebook entries bearing on the hypothesis, uncorrected.

5 Results

5.1 Lowering tracks concentration, not rain

Consecutive profiles were differenced to a lowering *rate* per station-fortnight rather than modelled as levels, which removes the pin-settlement artefact of long fixed-station records [5]. Log lowering rate was regressed on log load concentration — the Herfindahl index of the footfall distribution over 40 mm lateral bins — with total passages, rainfall, freeze–thaw days, soil moisture, season and instrument as covariates, in a mixed-effects model with station nested in site (20,983 station-fortnights at 84 stations on the four walked sites, ICC = 0.41). Estimates are in Table 2; the fitted relation is in Figure 5.

Lowering scales with concentration as a power law of exponent 1.83 (95% CI 1.71–1.95), as (1) requires. Rainfall enters at 0.004 per 10 mm ($p = 0.71$): a wet fortnight changes the lowering rate by some four parts in a thousand, which we cannot distinguish from nothing. Once concentration is in the model, weather does not predict the depth of the hole.

The negative-control outcome settles the obvious objection that the instrument is simply reporting rain in disguise. Lowering measured on the unwalked verge strip 600 mm outside the corridor behaves in exactly the opposite way — indifferent to concentration ($p = 0.83$), strongly responsive to rainfall ($p < 0.001$). The verge erodes when it rains. The path erodes when people are careful.

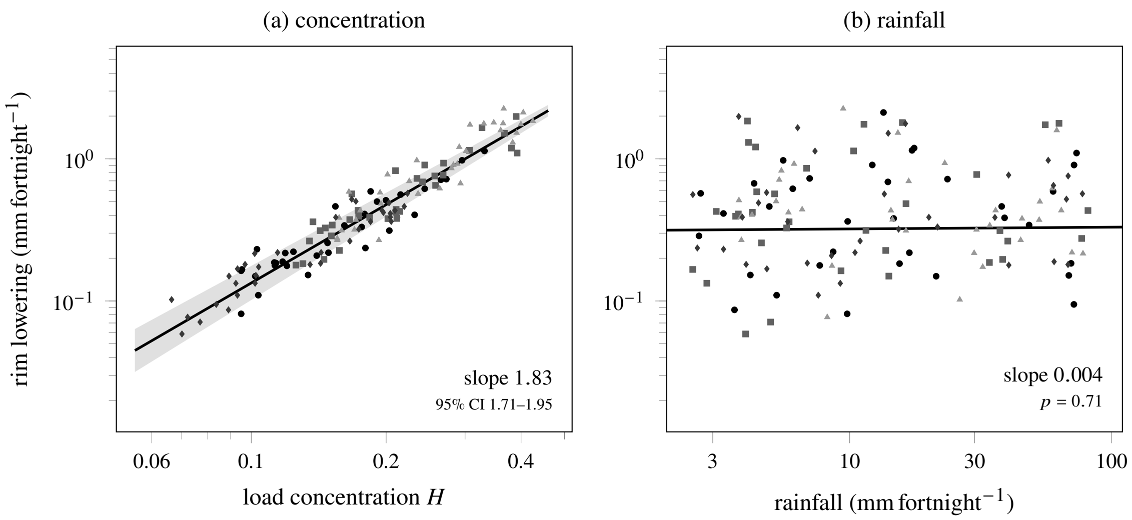

Figure 5: Concentration predicts the hole; rain does not. Rim lowering plotted against load concentration (a) and against fortnightly rainfall (b), on matched log axes, with sites marked separately.

Table 2: Log rim lowering rate per station-fortnight, mixed-effects estimates. Right-hand columns re-fit the same model to the unwalked verge as a negative-control outcome.

	Path rim		Verge (control)	
Predictor	Estimate	p	Estimate	p
log load concentration	1.83	< 0.001	0.01	0.83
Passages (per 1,000 d^{-1})	0.21	< 0.001	0.00	0.94
Rainfall (per 10 mm)	0.004	0.71	0.14	< 0.001
Freeze–thaw days	0.19	< 0.001	0.17	< 0.001
Soil moisture (% v/v)	0.02	0.55	0.03	0.41
Instrument (laser)	−0.06	0.62	−0.04	0.70

Two prespecified sensitivity analyses were run. Dropping Site C, which contributes most of the passage count, returns an exponent of 1.81; fitting to the post-2018 laser record alone returns 1.85. The rainfall coefficient stays at zero in both.

5.2 Water is neither necessary nor sufficient

The rainless interval. Between 3 May and 26 July 2022, Site C received no measurable rain for 84 days. The feature was dry throughout, its floor cracked, and it continued to deepen at 91 % of its wet-season rate. Traffic was unchanged. So was the offset distribution: walkers went on leaving a full berth around a dry depression they could have crossed without consequence. Water is not necessary.

The placebo path. Over the same eleven years, Site E — fenced, unwalked, same soil, same rainfall to within 4 % — lowered by 3 mm. Site A lowered by 210 mm. Water is not sufficient.

5.3 Manufacture of puddles on dry ground

If avoidance is the excavating agent, then the avoidance cue alone should suffice, and no water need ever be applied. We painted 24 matte blue-grey discs, 700 mm in diameter, on dry compacted hoggin at three sites, against 24 matched unpainted controls of identical geometry recorded only in the survey register. No water was applied to any plot at any time. Discs were installed as routine surface marking under the standing grounds-maintenance schedule; no walker was recruited, directed or recorded.

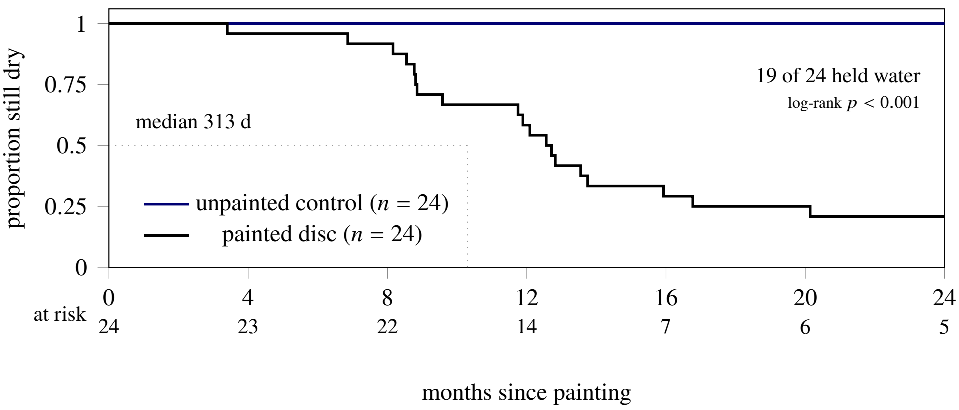

Figure 6: Twenty-four discs of paint on dry ground. Nineteen of them are now puddles.

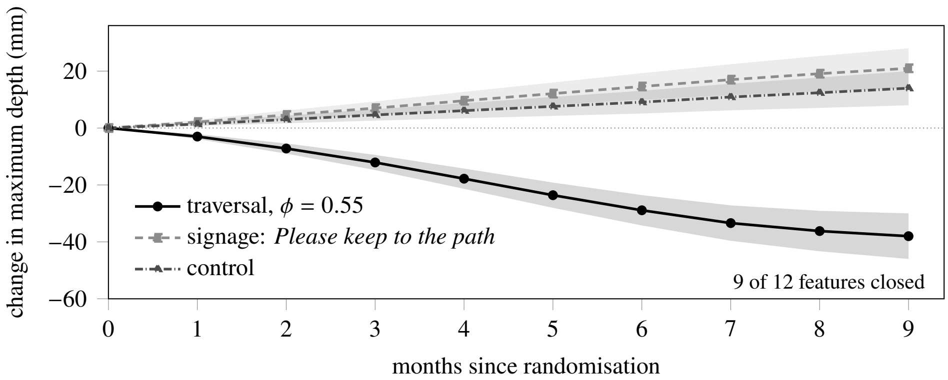

Figure 7: The randomised remediation trial. The arm that received a notice asking the public to keep to the path lost the most ground.

Nineteen of the 24 painted discs held standing water after rain by month 22. None of the 24 controls did (log-rank $p < 0.001$, stratified by site; Figure 6). Median time to first standing water was 313 days (IQR 271–366). Mean rim-ring lowering at 22 months was 41 mm on painted plots against 6 mm on controls. Two discs were painted over by contractors in February 2024; both had recorded standing water before that date and are retained, and the notebook entry marking them for exclusion (Figure 4) predates that check.

A painted disc is not a depression and holds no water on the day it is applied. It becomes one because it is walked around.

5.4 Randomised remediation

Thirty-six existing features across Sites A–D were randomised 1:1:1, with allocation concealed in sealed envelopes held by the parks directorate and outcome assessment performed on de-identified station codes. The traversal arm received a volunteer cohort instructed to walk straight through, raising measured ϕ from a baseline 0.11 to 0.55. The signage arm received the standard-practice comparator, a laminated notice reading *Please keep to the path*. The control arm received nothing. The primary outcome was change in maximum depth at nine months (Figure 7).

Arm	Δ depth (mm)	95% CI	Features closed
Traversal	−38	−46 to −30	9 of 12
Signage	21	+14 to +28	0 of 12
Control	14	+8 to +20	0 of 12

The primary comparison, traversal against control, is −52 mm ($p < 0.001$, Bonferroni-adjusted across the five prespecified outcomes). The secondary comparison, signage against control, is 7 mm ($p = 0.03$, adjusted). Signage did not fail to work. It worked, in the direction Corollary 2 specifies, and it was the fastest-growing arm in the trial.

5.5 The courtesy gradient

Across 61 paths in 14 authorities we measured ϕ and a *deference index*: the mean berth left in unobstructed two-way passing on dry surface, a purely behavioural quantity with no topographic input. Two results follow.

First, ϕ separates the paths almost perfectly. Of the 60 paths that could be classified, every one free of standing-water features had $\phi > \phi^*$ (minimum 0.38) and every one carrying a feature had $\phi < \phi^*$ (maximum 0.30). The sixty-first sits at $\phi = 0.33$ and has a feature that has closed and reopened twice since 2021; we take it to be on the boundary Proposition 2 predicts, and it is the only path we have found there.

Second, among the paths with features, growth rate correlates with the deference index at $\rho = 0.71$ (Pearson; 95% CI 0.56–0.82). The more courteous the traffic, the faster the hole.

6 Discussion

The standard account of the wet footpath has the causal arrow the wrong way round, and the whole of path management follows it. Drainage is installed to remove water from a depression the water did not make. Surfaces are dressed, which raises the yield threshold locally and moves the berth ring to the untreated ground beyond the dressing. Routes are widened, which by Corollary 3 supplies berth rather than removing the need for it. Notices are erected asking the public to keep to the path, and by Corollary 2 these are the most damaging instrument in routine use — a laminated sheet of A4 that removed more ground in nine months than nine months of unmanaged traffic.

The remedy is free and immediate. Above ϕ^* the feature closes, and ϕ^* is approximately one walker in three. A path carrying 400 people a day requires 136 of them to step in the puddle, and requires it of them only until the puddle is gone. Unprompted, the traffic does not do it: baseline ϕ across our sites is 0.11, and no site has drifted above ϕ^* without instruction. Our traversal arm reached 0.55 with a cohort of eleven volunteers on a rota.

The intervention is self-limiting. Once the feature has closed there is nothing left to step in, ϕ returns to its baseline, and the path re-enters the growth regime only when the next depression begins. Standing directives are therefore preferable to campaigns.

6.1 Limitations

Sealed asphalt footways develop depressions of the same form at 0.6 times the rate we observe on unsealed ground [3]. This should not happen. The yield threshold of a bituminous wearing course is two orders of magnitude above any pedestrian peak pressure our plate arrays have recorded, so L_c is unattainable there and Equation (1) returns exactly zero for every asphalt surface we have instrumented. We have repeated the calibration on three wearing courses and obtained the same nothing each time. The depressions are nonetheless there and we have measured them. We do not know what is producing them, and we have not been able to construct a version of this model that produces them without also predicting subsidence on surfaces where none occurs.

Three further limitations are more ordinary. The clearance ratio c was estimated only on treads between 0.9 m and 2.4 m wide; nothing here should be applied to corridors, staircases, or any geometry in which the walker cannot choose an offset freely. The traversal arm of §5.4 could not be blinded to participants, who necessarily knew whether they were walking through a puddle: outcome assessment was blinded, behaviour was not. And the courtesy gradient is cross-sectional. Deference and traversal were measured on the same paths at the same time, and a path that has acquired a large feature may teach its users to defer as readily as the reverse. The randomised trial fixes the direction for ϕ. It does not fix it for the deference index.

7 Conclusion

A puddle is not an obstruction that appeared on a path. It is a volume of ground removed from the path by everybody who declined to walk on it, and its width is the width they have given it. The excavating agent is the careful pedestrian. The rate is set by how careful they are. The water arrives afterwards and takes no part.

We therefore recommend that grounds staff be instructed, in writing, to walk through standing water on unsealed routes as a matter of routine, that traversal rotas be adopted where features exceed 1 m in diameter, and that the sign be taken down.

Acknowledgements

We thank the Netherfold parks staff for eleven years of access and for driving the pins, and the eleven members of the traversal rota. Footwear was replaced at the Survey's expense.

Data availability

Profilometry, gait-gate offsets and plate-array pressures are deposited with the Long Path Survey under accession LPS-2026-004. The painted-disc and remediation trials were pre-registered (LPS-PR-19, LPS-PR-24).

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