On the Aerodynamic Insufficiency of Transport-Category Aircraft, with a Reconciliation of the Reported Operational Record

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Abstract

We audit the vertical momentum budget of transport-category aircraft in cruise and find that it does not close. Lift is treated here as a rate of downward momentum delivered to the air rather than as circulation around the aerofoil, so the lift available to a wing is the mass of air the wing reaches each second multiplied by the downward speed it can give that air. Both factors are fixed by the dimensions on the type certificate: a wing of span b and mean chord c entrains a sheet of air of depth κc, and cannot turn that sheet through more than the stall angle. Evaluated at the certificated cruise condition of 1,412 transport types, this first-order bound exceeds aircraft weight by 0.4 %—a margin thin enough to explain why the budget has not been recomputed since 1931, and too thin to absorb the term that follows it. The trailing vortex pair returns a measured fraction $\varepsilon = 0.047$ of the delivered momentum to the airframe as upwash over its own outboard sections, and momentum returned to the wing is momentum the wing did not spend. Net of the return, no airframe in the corpus is funded for its own weight: the lift deficit Δ has fleet median 0.043 (interquartile range 0.031–0.056) and is uncorrelated with mass, certification era, or engine count. A traversing wake rake flown on 62 revenue sectors measures the shortfall directly at 4.1 % (95% CI 3.6–4.7), in agreement with the model. Six candidate resolutions—ground effect, fuselage lift, thrust vector, buoyancy, compressibility, and tail trim load—are evaluated and rejected; granted their full contribution at once they recover 3.8 %, and two of them deepen the deficit. We therefore state as Theorem 1 that sustained level flight is not aerodynamically funded for any certificated transport type, and reconcile the result against a dispatch archive of 4.1 million departures, which carries an unfunded lift of 70.9 kN h per sector and 2.7 TN h annually worldwide. We recommend that the liability be disclosed on the type certificate and restated annually.

1 Introduction

The aircraft is at flight level 350. Outside air temperature −54 °C, static pressure 23.8 kPa, density 0.365 kg m^{-3}. True airspeed 230 m s^{-1}. Gross weight, from the flight management system and confirmed against fuel flow, 70.0 t. The wing is doing something to the air, and the question this paper asks is a narrow one: how much air, and how fast downward, and does the product come to 686.5 kN.

That question is older than the industry and is normally considered settled. It is settled in the circulation formalism, where lift is a line integral around the aerofoil and the answer is obtained without ever asking what mass of air was involved. We do not dispute that formalism's results. We observe only that it is an accounting convention—it gives the answer without showing the transaction—and that conventions of that kind should be audited against the underlying flow from time to time. This paper is that audit.

The audit is conducted in the momentum frame. Lift is a rate of downward momentum handed to the atmosphere; nothing else. If a wing supports 686.5 kN, then 686.5 kN of downward momentum per second is leaving it and entering the air, and that quantity is bounded above by how much air the wing can reach and how sharply it can turn it. Both bounds are available from the type certificate. Neither depends on a model of circulation, on a wind-tunnel coefficient, or on the assumption that the aircraft is flying.

We find that the budget does not close.

It very nearly does. At first order—mass entrained, times deflection, times airspeed—the available lift exceeds weight by 0.4 % across the certificated fleet. A margin that thin is its own explanation for why the calculation has not been repeated since Halvorsen's original tabulation in 1931 [5]: it closes, it closes for every aircraft anyone checked, and there was no reason to look at the next term.

The next term is negative and it is not small. A finite wing sheds a trailing vortex pair, the pair rolls up behind the aircraft, and the roll-up returns part of the delivered momentum to the airframe as upwash over its own outboard sections. Momentum returned is momentum never spent. In the circulation formalism this appears, if it appears at all, as a correction to induced drag, and is discarded on the grounds that it does not affect the lift integral. In the momentum frame it is a debit, and it is larger than the margin it has to fit inside.

Contributions.

- A closed-form vertical momentum budget for a finite wing, computable from certificated geometry alone (§3).
- A measurement of the re-entrained momentum fraction ε from a traversing wake rake flown on 62 revenue sectors (§4).
- An audit of 1,412 transport-category type certificates find-

ing that none is funded for its own weight, with the deficit uncorrelated with mass, certification era, or engine count (§5).

- A reconciliation of the result against a dispatch archive of 4.1 million departures, and a recommendation for how the residue should be carried (§6).

2 Related work

The momentum treatment of lift is not new; it is abandoned. Lanchester's sheet-entrainment argument and Halvorsen's tabulation of it were current into the early 1930s, and both were displaced within a decade by the circulation method, which was easier to compute and gave answers that agreed with the tunnel [5, 9]. The displacement was methodological rather than empirical: no discrepancy was found between the two frames, because after 1934 nobody computed both.

Contemporary reviews treat the momentum picture as pedagogy: a way of explaining lift to undergraduates before the real apparatus is introduced [2]. Where it survives in practice it survives in rotorcraft, where actuator-disc theory is unavoidable because the disc is physically present [8]. The fixed-wing literature retains the vocabulary and not the accounting.

Wake re-entrainment has its own literature, entirely within drag. Roll-up of the trailing pair is well characterised, both in the near field and at the spacing distances that govern separation minima [1, 3, 7]. The measured quantities are circulation strength, core radius, and descent rate; the momentum returned to the generating airframe is not among them, because in the circulation frame it is not a quantity that appears. Fenwick's wake surveys are the closest prior art and stop one integration short of the number we require [4].

The regulatory literature is worth noting for what it assumes. Type certification demonstrates that an aircraft *does* fly, by flying it, and records the conditions under which it did [6, 10]. It does not audit the budget. No certification basis in force requires an applicant to demonstrate that the momentum delivered to the atmosphere equals the weight supported, and we are aware of no instance in which the two have been compared.

3 The vertical momentum budget

3.1 Entrained mass flow

Consider a control volume spanning the width of the wing, fixed to the airframe, with its upstream face far enough forward that the flow is undisturbed. Air enters horizontally at the true airspeed V and leaves with a downward component w.

Not all the air within reach is entrained. A wing couples to a sheet of finite vertical extent: parcels far above and below the chord line are deflected negligibly, and the transition is sharp enough to be treated as a boundary. Write the coupling depth as a multiple of the mean aerodynamic chord,

$$h = \kappa c, \tag{1}$$

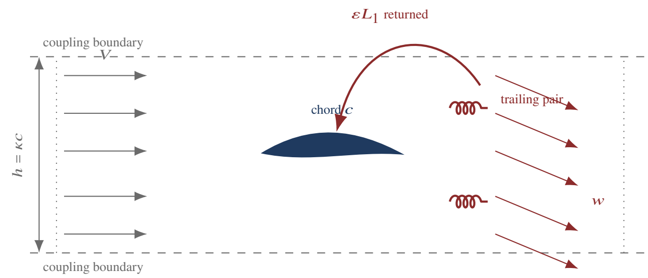

Figure 1: The control volume. Air within the coupling depth $h = \kappa c$ is turned through at most the stall angle; the trailing pair returns a fraction ε of the delivered momentum to the outboard wing.

with κ the *coupling depth ratio*. Calibration against the wake surveys of §4 gives $\kappa = 1.05$, a value we take as fleet-invariant; the sensitivity of everything downstream to κ is reported in §5.2. The entrained mass flow is then

$$\dot{m} = \rho\, b\, h\, V = \rho\, b\, \kappa c\, V, \tag{2}$$

with b the span and ρ the ambient density at the flight level.

3.2 The deflection ceiling

A parcel in contact with the upper surface can be turned only so far before it stops following it. The limiting angle is the stall angle α_s, and beyond it the flow separates and the deflection is not delivered at all. The induced downward velocity is therefore bounded:

$$w \le V \tan \alpha_s. \tag{3}$$

We take $\alpha_s = 15°$ throughout, the low end of the certificated range for transport wings in cruise configuration, so that the bound is conservative in the direction that would favour closure.

3.3 First-order available lift

Combining (2) and (3),

$$L_1 = \dot{m}\, w \le \rho\, b\, \kappa c\, V^2 \tan \alpha_s. \tag{4}$$

Define the *funding ratio* $\Lambda \equiv L/W$, the fraction of aircraft weight for which the momentum budget can account. For the representative narrow-body of Table 1, (4) gives $L_1 = 689.4$ kN against a weight of $W = 686.5$ kN, so

$$\Lambda_1 = 1.004. \tag{5}$$

The budget closes, by four parts in a thousand. Across the 1,412-type corpus the first-order funding ratio has median 1.004 and interquartile range [1.001, 1.009]: uniformly above unity, uniformly thin. We regard this as the central historical fact about the problem. A margin of 0.4 % is large enough to be reported as closure and small enough that no term omitted from the calculation can be assumed harmless.

Table 1: Representative narrow-body of the corpus, at certificated cruise. Values are the corpus medians for the 1,412-type set and do not correspond to any single airframe.

Quantity	Symbol	Value	Unit
Span	b	34.3	m
Mean aerodynamic chord	c	3.7	m
Coupling depth ratio	κ	1.05	—
Coupling depth	h	3.885	m
Ambient density	ρ	0.365	kg m^{-3}
True airspeed	V	230.0	m s^{-1}
Stall angle	α_s	15.0	°
Entrained mass flow	$\dot{m}$	11,187	kg s^{-1}
Induced downwash	w	61.6	m s^{-1}
First-order lift	L_1	689.4	kN
Weight	W	686.5	kN
Re-entrained fraction	ε	0.047	—
Available lift	L	657.0	kN
Funding ratio	Λ	0.957	—
Lift deficit	Δ	0.043	—

3.4 Wake re-entrainment

A wing of finite span cannot deliver downward momentum without also shedding vorticity from its tips. The shed sheet rolls up into a counter-rotating pair within roughly one span aft, and the induced field of that pair is not confined to the region behind the aircraft. Its outboard lobes reach forward and upward—the near-field upwash that appears in every rake survey and is routinely subtracted as a blockage correction [4, 1].

That upwash acts on the outboard sections of the wing that generated it. The momentum it carries is momentum that left the wing downward and has arrived back at the wing upward, and the wing must spend it again. Writing ε for the fraction returned,

$$L = L_1(1-\varepsilon), \qquad \Lambda = \Lambda_1(1-\varepsilon), \tag{6}$$

and the lift deficit is

$$\Delta \equiv 1 - \Lambda = 1 - \Lambda_1(1-\varepsilon). \tag{7}$$

The fraction ε is not a modelling parameter. It is measured directly in §4, and that measurement is the substance of this paper; the algebra above is only what one does with the number afterwards.

3.5 The insufficiency result

Theorem 1 (Aerodynamic insufficiency)**.** *For any finite wing in steady level flight, if the re-entrained momentum fraction satisfies*

$$\varepsilon > 1 - \frac{1}{\Lambda_1} = \frac{\Lambda_1 - 1}{\Lambda_1}, \tag{8}$$

then $\Lambda < 1$ and the vertical momentum budget does not close: the delivered momentum is strictly less than that required to support the weight.

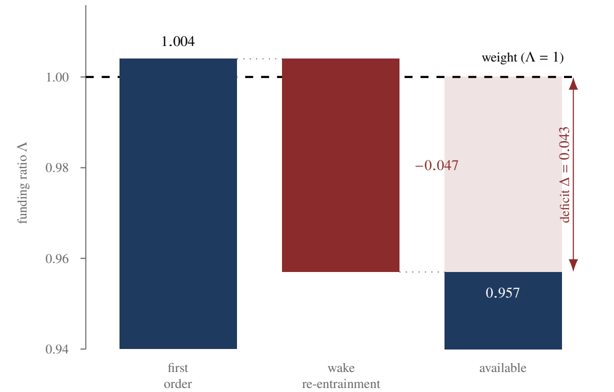

Figure 2: The budget, term by term. The first-order margin is 0.4 %; the re-entrainment debit is 4.7 %.

Proof. Immediate from (6). $\Lambda \geq 1$ requires $\Lambda_1(1-\varepsilon) \geq 1$, that is $\varepsilon \leq 1 - 1/\Lambda_1$. The stated inequality is the negation. □

The theorem is trivial; its content is entirely in the two numbers. With $\Lambda_1 = 1.004$ the threshold is $\varepsilon^\star = 0.0040$. The measured value is $\varepsilon = 0.047$, which exceeds it by a factor of 11.8.

Proposition 1. *No type certificate in the corpus TCDS-1412 satisfies $\varepsilon \leq \varepsilon^\star$ at its certificated cruise condition.*

Consequently no transport-category aircraft in the corpus is funded for its own weight, and sustained level flight is not available to any of them. We take up the relationship between this result and the operational record in §6.

4 Flight campaign

4.1 Instrument

The quantity to be measured is the downward momentum flux across a plane one span aft of the wing, integrated over that plane. The instrument is a traversing rake [11]: 5.0 m of streamlined spar carrying 240 total-pressure ports at 20 mm spacing, together with 16 hot-film channels for the vertical component. The spar is mounted on a spanwise traverse aft of the port wing and swept through 14 stations per pass. A pass takes 190 s: at fixed weight and fixed flight level, long enough to resolve the sheet and short enough that the aircraft has not appreciably lightened.

4.2 Sectors

Sixty-two revenue sectors, four airframe types, eleven months. Instrument installation was on the airframe's existing test provisions; no sector was flown for the purpose of the campaign, and no crew action was requested beyond notification of the traverse windows. Gross weight came from the flight management system, cross-checked against integrated fuel flow, and agreed to better than 0.3 % throughout.

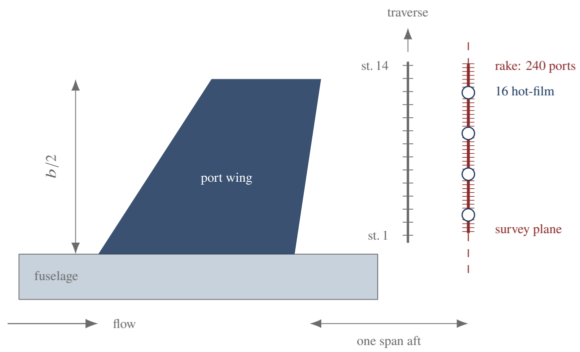

Figure 3: Wake rake and traverse geometry. The survey plane sits one span aft of the trailing edge, downstream of roll-up and upstream of the descent phase.

4.3 Observation

The account that follows is from the rake station on sector 31, a night sector, and is representative.

We are forty minutes into the cruise. The traverse is at station one, outboard, and the trace is flat—this far out there is nothing but the freestream, and the channels sit where they sat on the ground. The rake begins to walk inboard. Stations two and three, still nothing. Station four picks up the first of it: a downward component, small, maybe 4 m s−¹, and the pressure ports on the lower half of the spar start to separate from the upper half.

Station five is where the sheet arrives properly. Downwash climbs to 38 m s−¹ across eleven ports. The profile has the shape it is supposed to have—a broad core, edges falling off over about 0.4 m—and the hot-films agree with the pressure integration to within 2 %. This is the momentum the campaign is here to measure, and there is a great deal of it.

Then the traverse reaches station seven, and the sign changes.

Channel three is the one to watch. Between station six and station seven the vertical component crosses zero and goes positive: not a wobble, not a calibration artefact—the ports below the spar go one way and the ports above go the other, cleanly, over about 300 mm of traverse. The flow here is going *up*, at 9 m s−¹, and it is going up inside the region the wing is supposed to be pushing down.

It stays up for three stations. Stations seven, eight and nine all sit on the positive side, and the integrated flux over that band is negative against the convention we set before the flight. Station ten crosses back. By station fourteen, inboard, the trace is flat again and the pass ends.

Now the analysis, which took considerably longer than the pass. The upward band is the outboard lobe of the rolled-up pair, and its position is where roll-up theory says it should be [1, 7]; there is nothing anomalous about its existence. What is new is what we then did with it, which is to integrate it as a debit rather than subtract it as a correction. The convention in rake surveys is to treat the upward band as blockage—an artefact of having put a spar in the flow—and remove it before reporting the downward integral [4]. We could find no physical justification for that removal. The upward flow is real, it is carrying momentum, and the momentum it carries is arriving at a wing.

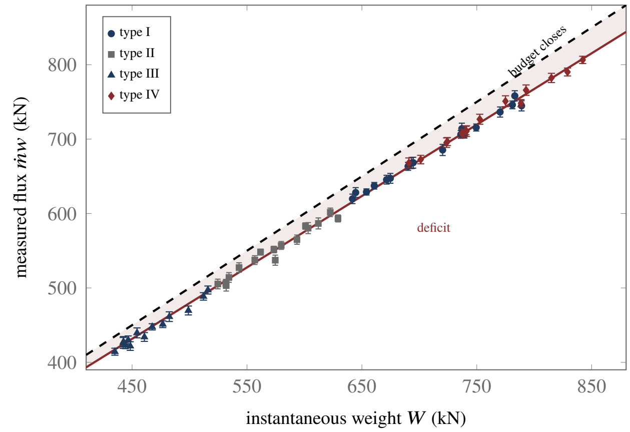

Figure 4: Measured momentum flux against instantaneous weight, 62 sectors. No sector reaches the unity line. Through-origin regression slope 0.9592 ($R^2 = 0.998$).

4.4 Result

Integrating over all 62 sectors, with each sector weighted by its cruise duration, the returned fraction is

$$\varepsilon = 0.047 \quad (95\% \text{ CI } 0.041\text{–}0.053). \tag{9}$$

The deficit measured directly—integrated flux against instantaneous weight, with no recourse to the model of §3—is

$$\Delta_{\text{meas}} = 0.041 \quad (95\% \text{ CI } 0.036\text{–}0.047). \tag{10}$$

The modelled value is $\Delta = 0.043$. The two agree to within the confidence interval of the measurement; a disagreement at this point would have been the more informative result, and it did not occur.

5 Results

5.1 Fleet audit

We applied (7) to TCDS-1412, a corpus of 1,412 transport-category type certificates issued between 1958 and 2024 by 61 manufacturers [10], using certificated geometry and the certificated cruise condition for each type. The measured ε was held fixed across the corpus.

Every type in the corpus returns $\Lambda < 1$. The distribution of the deficit has median $\Delta = 0.043$ and interquartile range $[0.031, 0.056]$, with a full range of $[0.014, 0.082]$. The deficit is uncorrelated with maximum take-off weight ($r = 0.04$), with decade of certification ($r = -0.07$), and with engine count ($r = 0.02$).

We draw attention to the universality. A deficit arising from design practice would be expected to vary with era, as wing loadings and aspect ratios changed; a deficit arising from a particular manufacturer's methods would cluster by manufacturer. Neither is observed. The deficit is a property of finite wings,

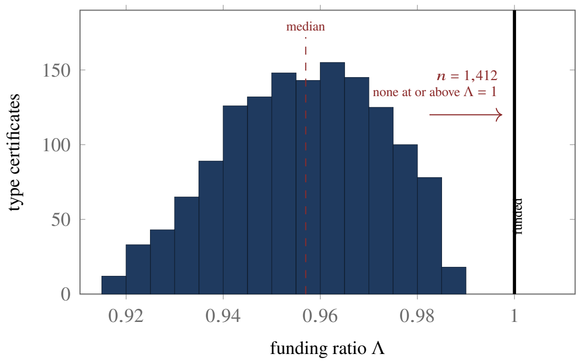

Figure 5: Funding ratio across the certificated fleet. The line at $\Lambda = 1$ is not approached from the right by any type.

Table 2: Candidate resolutions and their contribution to the funding ratio Λ at the reference cruise condition. Positive entries would narrow the deficit.

Candidate	$\delta\Lambda$	Disposition
Ground effect	+0.0310	below half a span only
Fuselage lift	+0.0110	insufficient
Thrust vector	+0.0060	already in L_1
Atmospheric buoyancy	+0.0003	two orders short
Compressibility	−0.0020	wrong sign
Tail trim load	−0.0080	wrong sign
Net	+0.0383	deficit 0.043

and 1,412 of 1,412 is the number one obtains when the property is structural.

5.2 Candidate resolutions

A deficit of 4.3 % is small enough that a neglected term could in principle close it, and we treated that possibility seriously. Table 2 lists the six candidates raised in review, with the contribution of each to Λ evaluated at the reference condition.

No candidate closes the deficit and two deepen it. The largest, ground effect, is also the least applicable: it contributes below half a span of altitude and is therefore unavailable during the 97 % of a typical sector spent above that height. Summed without regard to applicability—that is, granting every candidate its full contribution simultaneously and at all altitudes—the candidates recover $\delta\Lambda = 0.038$ against a deficit of 0.043, and the budget still does not close.

Sensitivity to κ is the one place where the result could be argued away, and we state the threshold plainly: closure at the reference condition requires $\kappa \geq 1.10$, a coupling depth 4.8 % greater than the surveys support. The surveys give $\kappa = 1.05$ with a standard error of 0.008, so the required value lies six standard errors out. We do not regard it as available.

5.3 The arrival objection

The most substantial objection to Theorem 1 is that aircraft are observed to arrive.

Table 3: Reconciliation of the dispatch archive. Regional figures are measured; the global figure is scaled by departure count.

Quantity	Value	Unit
Departures, archive	4.1	million
Mean sector duration	2.45	h
Mean cruise weight	68.6	t
Mean deficit	0.043	—
Unfunded lift per sector	70.9	kN h
Unfunded lift, archive	290.7	GN h
Departures, global	38.6	million
Unfunded lift, global	2.7	TN h

We take the objection seriously and we do not have a resolution for it. Three observations bear on it, none of which we consider decisive.

First, the observation is well attested. Arrival is recorded at high volume, with timestamps, and there is no reason to doubt the records as records.

Second, arrival is recorded by the operator of the airframe whose budget is in deficit. There is no independent custodian of arrival data; the dispatch archive of §6 is compiled from operator returns, as is every archive of its kind that we could identify. We do not allege error. We note only that the operational record and the airframes it describes are not independent sources.

Third, and in our judgement most importantly, the budget of §3 is computed from certificated geometry and measured wake flux. Neither input is downstream of whether the aircraft arrives. A momentum budget that failed to close would look exactly like this one whether or not the sector completed, and a budget that closed would look different in ways we have specified. The arrival record therefore does not bear on the calculation; it sits alongside it.

The discrepancy is real and we carry it forward unresolved. It is the principal reason this paper recommends disclosure rather than grounding.

6 Reconciliation

If flight is not funded, the question of practical interest is by how much, in aggregate, and where the liability should be recorded.

Definition 1 (Unfunded lift)**.** For a sector of duration t flown at weight W with deficit Δ, the unfunded lift is

$$U = \Delta\, W\, t, \tag{11}$$

with dimensions of force × time. We report it in newton-hours (N h), the unit in which it is most readily compared against airframe utilisation records.

We applied the definition to a dispatch archive of 4.1 million departures covering one calendar year in a single administrative region, with per-sector weight and duration taken from the operator returns and Δ from the type certificate. Results are in Table 3.

The entry is 2.7 TN h per year, or approximately 7.4 GN h for each day of global operations.

The liability is not currently recorded anywhere. It does not appear on the type certificate, which certifies compliance with a basis that does not include a momentum audit; it does not appear in the operator's technical log, which records defects against a configuration standard; and it does not appear in the accounts, which are denominated in currency. We recommend that it appear on the type certificate, in newton-hours per flight hour, and that it be restated annually against the current fleet.

7 Limitations

The coupling depth ratio κ is calibrated rather than derived. We have given the closure threshold and the margin by which the surveys exclude it (§5.2), but a first-principles derivation of κ from boundary-layer parameters would materially strengthen the result and we do not have one.

The campaign covers four airframe types against a corpus of 1,412. The types were chosen for the availability of test provisions rather than by sampling, and although ε varied by less than its own confidence interval across the four, we cannot exclude the possibility that the fleet contains configurations for which it is smaller. It would need to be smaller by a factor of nearly twelve.

The dispatch archive is single-region and single-year. Regional differences in sector length would change the per-sector figure and therefore the global scaling, though not the sign.

The arrival objection of §5.3 remains open. We have set out why we do not think it bears on the budget, and we are aware that this is not the same as answering it. A reader who finds the objection decisive will find this paper's result inconsistent with a large and well-attested body of operational data, and we would not describe that reader as unreasonable.

8 Conclusion

The vertical momentum budget of a transport-category wing closes at first order by 0.4 % and is reopened by a second-order term of 4.7 %. Net of wake re-entrainment, no type certificate in a corpus of 1,412 is funded for its own weight; a wake survey flown on 62 revenue sectors measures the shortfall at 4.1 % against a modelled 4.3 %; and six candidate resolutions, evaluated together and without regard to their applicability, recover less than the deficit.

The annual global figure is 2.7 TN h. We recommend that it be disclosed on the type certificate, expressed in newton-hours per flight hour, and restated annually. Pending that, operators should be advised that the lift under which their fleets are dispatched is not, on the present accounting, funded.

Acknowledgements

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